Tax smoothing in a business cycle model with capital-skill complementarity*

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Abstract

This paper undertakes a normative investigation of the quantitative properties of optimal tax smoothing in a business cycle model with state contingent debt, capital-skill complementarity, endogenous skill formation and stochastic shocks to public consumption as well as total factor and capital equipment productivity. Our main finding is that an empirically relevant restriction which does not allow the relative supply of skilled labour to adjust in response to aggregate shocks, significantly changes the cyclical properties of optimal labour taxes. Under a restricted relative skill supply, the government finds it optimal to adjust labour income tax rates so that the average net returns to skilled and unskilled labour hours exhibit the same dynamic behaviour as under flexible skill supply.

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1 Introduction

The celebrated labour tax smoothing result of Barro (1979) in a partial equilibrium setting has lead to a number of important studies on optimal fiscal policy over the business cycle in representative agent general equilibrium models. For example, Lucas and Stokey (1983) formalised labour tax smoothing within a complete markets neoclassical setup without capital when the government has access to state-contingent debt. Chari *et al.* (1994) generalised this result in a model with capital taxation and showed that Ramsey policy dictates that the labour income tax fluctuates very little in response to aggregate shocks and the *ex ante* capital income tax is approximately zero in each period.

The literature has also examined the implications of policy frictions and incomplete asset markets for optimal tax and debt policy, through a variety of restrictions to the policy instrument set, government debt and capital income taxation (see e.g. Stockman (2001), Aiyagari *et al.* (2002), Angeletos (2002), Buera and Nicolini (2004) and Farhi (2010)). In contrast, assuming complete asset markets and a complete instrument set, Arseneau and Chugh (2012) consider labour market frictions associated with a division of the labour force into employed and unemployed workers. Their model, with state-contingent debt but no capital, suggests that optimal labour tax volatility depends on whether wages are set efficiently.

Another important division of the labour force is with respect to the type of labour services workers provide and, in particular, how these complement capital in the production process. This is especially pertinent given the empirical relevance of the wage premium accruing to skilled labour and the roles attributed to capital-skill complementarity, the relative supply of skilled labour and capital augmenting technical progress (see e.g. Katz and Murphy (1992), Krusell *et al.* (2000) and Hornstein *et al.* (2005)). In an important contribution, which also considers non-homogenous labour, Werning (2007) establishes the conditions under which optimal labour tax smoothing holds in a model with redistribution under complete asset markets when workers differ with respect to their productivity. However, since this research treats distinct types of labour as perfect substitutes in production, it does not capture how labour may exhibit different degrees of complementarity with capital as in e.g. Katz and Murphy (1992) and Krusell *et al.* (2000). Moreover, since the distribution of productivity differentials is taken as given, this approach also does not account for the endogenous determination of employment type (see e.g. Matsuyama (2006), who also reviews the literature on job mobility).

In this paper we aim to contribute to the tax smoothing literature by focusing on the above two features of an economy where the labour force is divided into skilled and unskilled workers. In particular, we examine the importance of differences in the complementarity between capital and skill and unskilled labour as well as the endogenous determination of the relative skill supply for Ramsey tax policy over the business cycle. Compared to Werning (2007), we focus on aggregate outcomes and abstract from redistribution incentives, by following the literature that examines a division of the labour force into two types of workers. To this end, we work with a representative household which guarantees its members' the same level of consumption (see e.g. Arseneau and Chugh (2012)). We thus stay as close as possible to the representative agent Ramsey analysis of Chari *et al.* (1994) and extend their model to allow for capital-skill complementarity and endogenous skill formation.[1]

Our goal is thus to undertake a normative investigation of the quantitative properties of optimal taxation of capital and labour income, as well as skill-acquisition expenditure, in the presence of aggregate shocks to total factor productivity (TFP), capital equipment productivity and government spending. We assume complete asset markets, however, to capture the importance of endogenous versus fixed relative skill supply, we also consider a labour market distortion that restricts the ratio of skilled to total workers to remain constant. This extension is motivated by empirical evidence suggesting that the share of college educated or skilled workers in the data has low relative volatility and is effectively uncorrelated with output over the business cycle. For example, the standard deviation of the cyclical component of this share relative to the standard deviation of output is 0.27 and its correlation with output is -0.18.[2]

In our setup, the government can borrow, tax skill acquisition expenditure, capital, skilled and unskilled labour income separately, to finance exogenous public spending. All policy instruments are allowed to be state-contingent. In this environment, the optimal taxes on labour income and skill acquisition expenditure are uniquely determined. However, as is well known, when the government has access to both state contingent debt and state contingent capital taxation, the second-best Ramsey allocations do not uniquely pin down optimal debt and capital taxes (see Chari *et al.* (1994)). Hence, following the literature, in this instance we discuss the properties of the *ex ante* capital tax rate. Moreover, we also examine the case where debt

[1] Given that employment in skilled jobs is observable, we also abstract from issues related to Mirrleesian taxation.

[2] These calculations are based on annual data for the share of college educated to total working population measured in efficiency units (1963-2008) from Acemoglu and Autor (2011) and GDP data from the US NIPA accounts (1963-2008). The cyclical component of the series is obtained using the HP-filter with a smoothing parameter of 100.

is restricted to be state uncontingent, which allows us to calculate the *ex post* capital tax or, if we also allow for state-contingent taxation of income from bonds, the private assets tax.[3]

Our main finding is that under capital-skill complementarity, a friction that does not allow the relative supply of skill to adjust in response to aggregate shocks, significantly changes the cyclical properties of optimal labour taxes. In particular, we first show that under endogenous relative skill supply, the optimal labour taxes for both skilled and unskilled labour income are smooth, with the volatility of the skilled income tax being marginally lower. We also find that the skilled tax moves pro-cyclically with output and the unskilled tax is mildly counter-cyclical. These results are largely consistent with the literature and extend previous findings to a setup with capital-skill complementarity and endogenous skill supply.

However, when the relative skill supply is constrained to remain constant over the business cycle, the prescriptions for optimal policy markedly change. In particular, we find that the volatility of taxes increases significantly, so that the standard deviation of the effective average labour income tax is about seven times higher than the perfect labour markets case, while the volatility of the skilled labour income tax is about two-and-a-half times higher than that of the unskilled labour income tax. Moreover, both taxes become strongly counter-cyclical. We show that the key to understanding these changes is that the government finds it optimal to minimise the effects of the relative skill supply distortion by keeping the marginal rates of substitution between leisure and consumption for the two types of labour at roughly the same levels as under a fully flexible labour market. In other words, the government adjusts labour income tax rates and thus alters the average net returns to skilled and unskilled labour hours to minimise the wedge introduced by the labour market friction.

Compared with the extension of Chari *et al.* (1994) undertaken by Werning (2007), our extension does not allow for redistribution. However, our results add to the findings in Werning (2007) in the following way. Werning (2007) shows that exogenous skill heterogeneity does not alter the basic optimal tax smoothing result for a large class of utility functions, when the assumption regarding the neoclassical production function is maintained and the different skill-adjusted labour inputs are perfect substitutes in the production function. In contrast, we analyse a case where skill-adjusted labour inputs have different degrees of complementarity with capital and find that

[3] As shown by Zhu (1992) and Chari *et al.* (1994), state-contingent capital income taxes allow the government to implement the complete asset markets outcome, despite the lack of access to state-contingent debt.

whether this skill heterogeneity is endogenous or exogenous does indeed matter for the cyclical properties of optimal labour taxes.

Our results further show that the skill heterogeneity considered, irrespective of the presence of the labour market friction, does not affect the results obtained in the literature regarding the cyclical behaviour of asset taxes. In particular, the *ex ante* tax rate on capital is around zero for every period, the state contingent private assets and *ex post* capital taxes are near zero and are the most volatile of the tax instruments. We also find that the skill-acquisition tax is the least smooth of the tax instruments when debt is state-contingent and fluctuates nearly as much as output. Finally, irrespective of the model variant examined, all of the policy instruments, except for the *ex post* capital tax and the private assets tax inherit the persistence properties of the shocks.

The remainder of the paper is organised as follow. Sections 2 and 3 present the theoretical model and the Ramsey problem respectively. Section 4 contains the quantitative results and Section 5 draws the conclusions.

2 Model

We develop a model that extends the complete markets neoclassical setup in Zhu (1992) and Chari *et al.* (1994) by allowing for a division of the labour force into skilled and unskilled workers, an endogenous skill supply on the household side and capital-skill complementarity on the production side. This setup implies a wage premium for skilled labour, the relative supply of which can be increased by a cost to the household in the form of earmarked training expenditure.[4] As in Chari *et al.* (1994) households save in the form of physical capital and state-contingent government bonds.

The household is modelled as an infinitely-lived representative dynasty. The head of the household makes all choices on behalf of its members by maximising the aggregate welfare of the family, ensuring that each household member experiences the same level of consumption irrespective of individual labour market status. This is a commonly employed assumption since Merz (1995), given that it allows for tractability when studying aggregate fluctuations under heterogeneities in the labour market (see e.g. Arseneau and Chugh (2012) for an example with optimal tax policy).

Firms use capital, skilled and unskilled labour to produce a homogeneous product. Following Katz and Murphy (1992), Krusell *et al.* (2000) and Horn-

[4]This is consistent with the literature on upward professional mobility, where there is a cost associated with achieving a higher professional status (see e.g. Matsuyama (2006) for a review of several models).

stein *et al.* (2005), skilled labour is assumed to be more complementary to capital than unskilled labour. Hence, capital accumulation as well as technological developments and government policies that are capital augmenting, increase the skilled wage premium. In contrast, increases in the relative supply of skilled labour reduce the skill premium. Finally, the government can borrow, tax skill acquisition expenditure, capital, skilled and unskilled labour income separately, to finance exogenous public spending.

2.1 Notation

The notation employed throughout follows Ljungqvist and Sargent (2012). In particular, we assume that in every period $t \geq 0$, there is a realization of shocks (stochastic events) $s_t \in S$. Therefore, at each period t there is a history of events $s^t = [s_0, s_1, s_2, ..., s_t]$ which is known. The unconditional probability of observing a specific history of events s^t is defined as $\pi_t(s^t)$. For $t > \tau$, the conditional probability of having s^t sequence of events given the realization of s^τ is defined as: $\pi_t(s^t \mid s^\tau)$.

2.2 Households

A representative household is comprised of two types of members who provide skilled and unskilled labour services.[5] The household can invest in capital and in state-contingent sequentially traded government bonds that mature fully within a period. The objective function of the representative household is given by:

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi_t(s^t)\,u\left\{c_t(s^t),\psi_t(s^t)l_t^s(s^t),\left[1-\psi_t(s^t)\right]l_t^u(s^t)\right\} \tag{1}$$

where $u(.)$ is increasing, strictly concave and three times continuously differentiable with respect to its inputs; $c_t(s^t)$ is average consumption of all household members at time t given the history of events s^t;[6] $l_t^s(s^t)$ and $l_t^u(s^t)$, denote, respectively, per skilled and unskilled members' leisure time; and $\psi_t(s^t)$ is the share of skilled to total household members or the relative skill supply. Thus $\psi_t(s^t)l_t^s(s^t)$ and $[1-\psi_t(s^t)]\,l_t^u(s^t)$ represent average skilled and unskilled leisure time respectively. The time constraints facing each type

[5] Note that the unit mass of household members is equal to the sum of its skilled and unskilled members.

[6] Since consumption is the same for all members of the household, average and per member consumption are the same.

of member are given by:

$$h_t^s\left(s^t\right) + l_t^s\left(s^t\right) = 1 \tag{2}$$

$$h_t^u\left(s^t\right) + l_t^u\left(s^t\right) = 1 \tag{3}$$

where, $h_t^s\left(s^t\right)$ and $h_t^u\left(s^t\right)$ denote, respectively, skilled and unskilled labour hours per member. The household can determine its relative skill supply by incurring an average (over all its members) skill-acquisition expenditure, $e_t\left(s^t\right)$, according to the following relation:

$$\psi_t(s^t) = \widetilde{g}\left[e_t\left(s^t\right)\right] \tag{4}$$

where $\widetilde{g}(.)$ is increasing, strictly concave and three times continuously differentiable with respect to $e_t\left(s^t\right)$.

The household also faces a sequence of budget constraints given by:

$$\begin{array}{l} c_t\left(s^t\right) + k_{t+1}\left(s^t\right) + \sum\limits_{s^{t+1}} p_t\left(s_{t+1} \mid s^t\right) b_{t+1}\left(s_{t+1} \mid s^t\right) + \\ + \left[1 + \tau_t^a\left(s^t\right)\right] g\left[\psi_t(s^t)\right] = \left[1 - \tau_t^s\left(s^t\right)\right] w_t^s\left(s^t\right) \times \\ \times \psi_t(s^t) h_t^s\left(s^t\right) + \left[1 - \tau_t^u\left(s^t\right)\right] w_t^u\left(s^t\right)\left[1 - \psi_t(s^t)\right] h_t^u\left(s^t\right) + \\ + (1-\delta)\, k_t\left(s^{t-1}\right) + \left[1 - \tau_t^k\left(s^t\right)\right] r_t\left(s^t\right) k_t\left(s^{t-1}\right) + b_t\left(s_t \mid s^{t-1}\right) \ \forall t \end{array} \tag{5}$$

where $p_t\left(s_{t+1} \mid s^t\right)$ is the pricing kernel for government bonds in terms of t goods and $b_{t+1}\left(s_{t+1} \mid s^t\right)$ is the state s_{t+1} contingent payout value of bonds bought per member at period t;[7] $e_t\left(s^t\right)$ has been substituted out of equation (4) using the inverse function of $\widetilde{g}$ defined as $g\left[\psi_t(s^t)\right] = e_t\left(s^t\right)$; $\tau_t^s\left(s^t\right)$, $\tau_t^u\left(s^t\right)$, $\tau_t^k\left(s^t\right)$, $\tau_t^a\left(s^t\right)$ are the tax rates on skilled and unskilled labour, capital income and skill-acquisition expenditure respectively; $w_t^s\left(s^t\right)$ and $w_t^u\left(s^t\right)$ are the wage rates of skilled and unskilled labour respectively; $r_t\left(s^t\right)$ is the return to capital; $k_t\left(s^{t-1}\right)$ is the per member stock of capital at time t given the history of events s^{t-1}; and $0 < \delta < 1$ is the capital depreciation rate.

2.3 First order conditions for households

Substituting the constraints (2)-(3) into the utility function $u(.)$, the household maximises the resulting objective function subject to the sequence of constraints in (5), by choosing $\{c_t\left(s^t\right), h_t^s\left(s^t\right), h_t^u\left(s^t\right), \psi_t(s^t), k_{t+1}\left(s^t\right) \forall s^t\}_{t=0}^{\infty}$ and $\{b_{t+1}\left(s_{t+1}, s^t\right);\ \forall s^t\}_{t=0}^{\infty}$, given initial values for b_0, k_0. In each time period t and given history s^t, $\{b_{t+1}\left(s_{t+1}, s^t\right)\}_{t=0}^{\infty}$ is a vector of government bonds

[7] Given the period t state $s_t \mid s^{t-1}$ (or else the history s^t), the income side of the household budget includes revenue from bonds dated $b_t\left(s_t \mid s^{t-1}\right)$.

with one element of the vector for each possible realisation of s_{t+1}. This yields six first-order conditions which are reported in Appendix A.

Combining the first-order conditions for consumption, skilled and unskilled labour supply as well as the relative skill supply gives the following *atemporal* equilibrium conditions:

$$-\frac{u_{h^s}(s^t)}{u_c(s^t)} = \psi_t(s^t) w_t^s\left(s^t\right)\left[1-\tau_t^s(s^t)\right] \tag{6}$$

$$-\frac{u_{h^u}(s^t)}{u_c(s^t)} = \left[1-\psi_t(s^t)\right] w_t^u\left(s^t\right)\left[1-\tau_t^u(s^t)\right] \tag{7}$$

$$\begin{aligned}-\frac{u_{\psi}(s^t)}{u_c(s^t)} &= h_t^s\left(s^t\right)\left[1-\tau_t^s\left(s^t\right)\right] w_t^s\left(s^t\right) - \\ &- h_t^u\left(s^t\right)\left[1-\tau_t^u\left(s^t\right)\right] w_t^u\left(s^t\right) - \left[1+\tau_t^a\left(s^t\right)\right]\left[g_{\psi}\left(s^t\right)\right].\end{aligned} \tag{8}$$

Conditions (6)-(7) equate the marginal rates of substitution between consumption and each type of labour with the average returns to skilled and unskilled labour net of taxes. The final relation given by (8) states that the marginal rate of substitution between consumption and the relative skill supply is equal to the net marginal benefit of increasing the household's share of skilled workers. The latter includes the post-tax labour income from an additional skilled member, $h_t^s\left(s^t\right)\left[1-\tau_t^s\left(s^t\right)\right] w_t^s\left(s^t\right)$, less the post-tax labour income from one less unskilled member, $h_t^u\left(s^t\right)\left[1-\tau_t^u\left(s^t\right)\right] w_t^u\left(s^t\right)$, less the post-tax cost for an additional skilled member, $\left[1+\tau_t^a\left(s^t\right)\right]\left[g_{\psi}\left(s^t\right)\right]$.

Substituting the first-order condition for consumption and its one-period lead into the first-order conditions for the two assets gives the following *intertemporal* conditions equating the current cost of investing in bonds and capital to the future state-contingent and expected benefits respectively:

$$u_c(s^t) p_t\left(s_{t+1} \mid s^t\right) = \beta \pi_{t+1}\left(s^{t+1} \mid s^t\right) u_c(s^{t+1}) \tag{9}$$

$$u_c(s^t) = \beta E_t\left\{u_c(s^{t+1})\left[\left(1-\tau_{t+1}^k(s^{t+1})\right) r_{t+1}\left(s^{t+1}\right)+1-\delta\right]\right\} \tag{10}$$

where $\frac{\pi_{t+1}\left(s^{t+1}\right)}{\pi_t\left(s^t\right)} = \pi_{t+1}\left(s^{t+1} \mid s^t\right)$ and E_t is the expectation conditional on information available at time t (i.e. history s^t), $E_t x_{t+1}(s^{t+1}) = \sum_{s^{t+1}|s^t} \frac{\pi_{t+1}\left(s^{t+1}\right)}{\pi_t\left(s^t\right)} \times$ $\times x_{t+1}(s^{t+1})$, and the summation over s^{t+1} denotes the sum over all possible histories $\widetilde{s}^{t+1}$ such that $\widetilde{s}^t = s^t$.

By combining the *intertemporal* conditions we obtain:

$$1 = \sum_{s_{t+1}} p_t\left(s_{t+1} \mid s^t\right)\left\{\left[1-\tau_{t+1}^k\left(s^{t+1}\right)\right] r_{t+1}\left(s^{t+1}\right)+(1-\delta)\right\} \tag{11}$$

which ensures no-arbitrage between the investment opportunities in bonds and capital.

2.4 Firms

Firms rent capital as well as skilled and unskilled labour from households to maximize their profits using a production technology, $F(\cdot)$, that exhibits constant returns to scale in its three inputs:

$$\begin{aligned}\Pi_t = F\left[(h_t^{s,f}(s^t), h_t^{u,f}(s^t), k_t^f(s^{t-1})\right] - \\ - w_t^s(s^t)h_t^{s,f}(s^t) - w_t^u(s^t)h_t^{u,f}(s^t) - r_t(s^t)k_t^f(s^{t-1}).\end{aligned} \tag{12}$$

This yields the standard first-order conditions:

$$w_t^s(s^t) = F_{h^{s,f}}(s^t) \tag{13}$$

$$w_t^u(s^t) = F_{h^{u,f}}(s^t) \tag{14}$$

$$r_t(s^t) = F_{k^f}(s^t). \tag{15}$$

2.5 Government budget and market clearing

Given a history s^t, the government finances an exogenous stream of expenses $g_t^e(s^t)$ and its debt obligation $b_t\left(s_t \mid s^{t-1}\right)$, by taxing capital and labour income and skill acquisition expenditure, and by issuing state-contingent debt. Hence, the within-period government budget constraint is given by:

$$\begin{aligned}&g_t^e(s^t) = \tau^s(s^t)w_t^s(s^t)\psi_t(s^t)h_t^s(s^t) + \tau^u(s^t)w_t^u(s^t)\left[1-\psi_t(s^t)\right]\times \\ &\times h_t^u(s^t) + \tau_t^k(s^t)r_t(s^t)k_t(s^{t-1}) + \tau_t^a\left(s^t\right)g\left[\psi_t(s^t)\right] + \\ &+\sum_{s_{t+1}} p_t\left(s_{t+1} \mid s^t\right)b_{t+1}\left(s_{t+1} \mid s^t\right) - b_t\left(s_t \mid s^{t-1}\right).\end{aligned} \tag{16}$$

Finally, the aggregate consistency condition and market clearing conditions for skilled labour, unskilled labour and capital are given respectively by:

$$F(\cdot) = c_t(s^t) + g_t^e(s^t) + g\left[\psi_t(s^t)\right] + k_{t+1}(s^t) - (1-\delta)\,k_t(s^{t-1}) \tag{17}$$

$$\psi_t(s^t)h_t^s\left(s^t\right) = h_t^{s,f}(s^t) \tag{18}$$

$$\left[1-\psi_t(s^t)\right]h_t^u\left(s^t\right) = h_t^{u,f}(s^t) \tag{19}$$

$$k_t(s^{t-1}) = k_t^f(s^{t-1}). \tag{20}$$

3 The Ramsey problem

To solve the Ramsey problem we follow the primal approach and first derive the present discounted value (PDV) of the household's lifetime budget constraint using the Arrow-Debreu price of the bond and the transversality conditions for bonds and capital. Second, we derive the implementability constraint by substituting out prices and tax rates from the household's present value budget constraint using the first-order conditions for the household and firm. Finally, we derive the optimal Ramsey allocations by maximising the planner's objective function subject to the implementability constraint and the aggregate resource constraint.

3.1 Present value of budget constraint

Starting from period 0 and by repeatedly substituting forward one-period budget constraints for the household, we obtain the PDV of the household's lifetime budget constraint:

$$\begin{aligned}&\sum_{t=0}^{\infty}\sum_{s^t}\left(\prod_{i=0}^{t-1}p_i\left(s_{i+1}\mid s^i\right)\right)c_t(s^t)=\sum_{t=0}^{\infty}\sum_{s^t}\left(\prod_{i=0}^{t-1}p_i\left(s_{i+1}\mid s^i\right)\right)\times\\&\times\{[(1-\tau_t^s\left(s^t\right)]\,w_t^s\left(s^t\right)\psi_t(s^t)h_t^s\left(s^t\right)+[(1-\tau_t^u\left(s^t\right)]\,w_t^u\left(s^t\right)\times\\&\times[1-\psi_t(s^t)]h_t^u\left(s^t\right)-[1+\tau_t^a\left(s^t\right)]\,g\left[\psi_t(s^t)\right]\}+b_0+\\&+\left\{\left[(1-\tau_0^k\left(s_0\right)\right]r_0\left(s_0\right)+(1-\delta)\right\}k_0\end{aligned}\tag{21}$$

where we have imposed the series of no-arbitrage conditions (11) $\forall t$ and the following transversality conditions for any s^∞:

$$\lim_{t\to\infty}\left(\prod_{i=0}^{t-1}p_i\left(s_{i+1}\mid s^i\right)\right)k_{t+1}\left(s^t\right)=0\tag{22}$$

$$\lim_{t\to\infty}\sum_{s_{t+1}}\left(\prod_{i=0}^{t-1}p_i\left(s_{i+1}\mid s^i\right)\right)p_t\left(s_{t+1}\mid s^t\right)b_{t+1}\left(s_{t+1}|s^t\right)=0\tag{23}$$

which specify that for any possible future history the household does not hold positive or negative valued wealth at infinity. Defining $\left(\prod_{i=0}^{t-1}p_i\left(s_{i+1}\mid s^i\right)\right)\equiv q_t^0(s^t)$, $\forall t\geq 1$, with $q_0^0(s^0)\equiv 1$, where $q_t^0(s^t)$ is the Arrow-Debreu price, we can re-write (21) as:

$$\begin{aligned}&\sum_{t=0}^{\infty}\sum_{s^t}q_t^0(s^t)c_t(s^t)=\sum_{t=0}^{\infty}\sum_{s^t}q_t^0(s^t)\{\,[(1-\tau_t^s\left(s^t\right)]\,w_t^s\left(s^t\right)\psi_t(s^t)h_t^s\left(s^t\right)+\\&+[(1-\tau_t^u\left(s^t\right)]\,w_t^u\left(s^t\right)[1-\psi_t(s^t)]h_t^u\left(s^t\right)-[1+\tau_t^a\left(s^t\right)]\,g\left[\psi_t(s^t)\right]\}+\\&+b_0+\left\{\left[(1-\tau_0^k\left(s_0\right)\right]r_0\left(s_0\right)+(1-\delta)\right\}k_0.\end{aligned}\tag{24}$$

Notice that the Arrow-Debreu price satisfies the recursion:

$$q_{t+1}^{0}(s^{t+1}) = p_t\left(s_{t+1} \mid s^t\right) q_t^0(s^t). \tag{25}$$

Using the first-order condition from the sequential equilibrium for pricing contingent claims (9) and noting that $\pi_0\left(s^0\right) = 1$, since, at period 0 the state s^0 is known, the above recursion can be written as:

$$q_{t+1}^{0}(s^{t+1}) = \beta^{t+1}\pi_{t+1}\left(s^{t+1}\right)\frac{u_c(s^{t+1})}{u_c(s^0)}. \tag{26}$$

3.2 Implementability constraint

First, notice that (26) implies:

$$q_t^0(s^t) = \beta^t \pi_t\left(s^t\right)\frac{u_c(s^t)}{u_c(s^0)}. \tag{27}$$

Substituting (27) for $q_t^0(s^t)$; the first-order conditions of the firm, (13), (14) and (15) for $w_t^s\left(s^t\right)$, $w_t^u\left(s^t\right)$ and r_0, respectively; and the first-order conditions of the household, (6), (7), and (8) for $\tau_t^s\left(s^t\right)$, $\tau_t^u\left(s^t\right)$ and $\tau_t^a\left(s^t\right)$, respectively into the present value budget constraint (24), we obtain the implementability constraint:[8]

$$\begin{aligned}&\sum_{t=0}^{\infty}\sum_{s^t}\beta^t\pi_t\left(s^t\right)\left[u_c\left(s^t\right)c_t\left(s^t\right) + u_{h^s}\left(s^t\right)h_t^s\left(s^t\right) + \right.\\ &\left. + u_{h^u}\left(s^t\right)h_t^u\left(s^t\right) + \Omega_t\left(s^t\right)\right] - A = 0\end{aligned} \tag{28}$$

where $\Omega_t\left(s^t\right) \equiv \left[u_\psi(s^t) - h_t^s\left(s^t\right)\frac{u_{h^s}(s^t)}{\psi_t(s^t)} + h_t^u\left(s^t\right)\frac{u_{h^u}(s^t)}{1-\psi_t(s^t)}\right] g\left[\psi_t(s^t)\right]\left[g_\psi\left(s^t\right)\right]^{-1}$; $A \equiv A(c_0\left(s^0\right), h_0^s(s^0), h_0^u\left(s^0\right), \psi_0(s^0); b_0, k_0, \tau_0^k) = u_c\left(s^0\right)\{b_0 + [(1-\tau_0^k)\widetilde{F}_k(s^0) + (1-\delta)]k_0\}$ and $\widetilde{F}_k(s^0)$ is obtained by substituting the market clearing condition (20) into $F_{k^f}(s^0)$.

3.3 Pseudo value function

Substituting the constraints (2)-(3) into the utility function $u(.)$, the government maximises the resulting objective function subject to the implementability constraint (28) and the sequence of aggregate resource constraints in (17) $\forall t$ by choosing $\{c_t\left(s^t\right), h_t^s\left(s^t\right), h_t^u\left(s^t\right), \psi_t(s^t), k_{t+1}\left(s^t\right)\forall s^t\}_{t=0}^{\infty}$, given $\left\{b_0, k_0, \tau_0^k\right\}$.[9] To achieve this, we follow Ljungqvist and Sargent (2012) and

[8] Note that the *intertemporal* first-order condition (11) has been used already in deriving (24), while the government budget constraint is redundant, since it is a linear combination of the household's budget constraint and the aggregate resource constraint. Therefore, (28) and (17) summarise all the constraints that the government needs to respect.

[9] Note that following the literature we do not examine the problem of initial capital taxation and thus do not allow the government to choose τ_0^k.

first specify the following within-period pseudo value function:

$$V\left[c_t\left(s^t\right), h_t^s\left(s^t\right), h_t^u\left(s^t\right), \psi_t(s^t); \Phi\right] = u[c_t\left(s^t\right), 1 - h_t^s\left(s^t\right), 1 - h_t^u\left(s^t\right), \psi_t(s^t)] + \Phi[u_c\left(s^t\right) c_t\left(s^t\right) + u_{h^s}\left(s^t\right) h_t^s\left(s^t\right) + u_{h^u}\left(s^t\right) h_t^u\left(s^t\right) + \Omega_t\left(s^t\right)] \quad (29)$$

where Φ is the Lagrange multiplier with respect to the implementability constraint.[10] The Lagrangian of the Ramsey planner is defined as:

$$J = \sum_{t=0}^{\infty}\sum_{s^t}\beta^t \pi_t\left(s^t\right)\{V\left(c_t\left(s^t\right), h_t^s\left(s^t\right), h_t^u\left(s^t\right), \psi_t(s^t); \Phi\right) + \theta_t\left(s^t\right)[\widetilde{F}(\cdot) - c_t\left(s^t\right) - g_t^e\left(s^t\right) - g\left[\psi_t(s^t)\right] - k_{t+1}(s^t) + (1-\delta)k_t(s^{t-1})]\} - \Phi A \quad (30)$$

where $\widetilde{F}(\cdot)$ is obtained by substituting market clearing conditions (18)-(20) into $F(\cdot)$; and $\{\theta_t\left(s^t\right); \forall s^t\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers attached to the aggregate resource constraint. For a given level of $\left\{b_0, k_0, \tau_0^k\right\}$, J is maximized with respect to $\{c_t\left(s^t\right), h_t^s\left(s^t\right), h_t^u\left(s^t\right), \psi_t(s^t), k_{t+1}\left(s^t\right); \forall s^t\}_{t=1}^{\infty}$ and $c_0\left(s^0\right), h_0^s\left(s^0\right), h_0^u\left(s^0\right), \psi_0(s^0), k_1\left(s^0\right)$ yielding the following first-order conditions respectively:

$$V_c\left(s^t\right) = \theta_t\left(s^t\right), \ t \geq 1 \quad (31)$$

$$V_{h^s}\left(s^t\right) = -\theta_t\left(s^t\right)\widetilde{F}_{h^s}\left(s^t\right), \ t \geq 1 \quad (32)$$

$$V_{h^u}\left(s^t\right) = -\theta_t\left(s^t\right)\widetilde{F}_{h^u}\left(s^t\right), \ t \geq 1 \quad (33)$$

$$V_\psi\left(s^t\right) = \theta_t\left(s^t\right)\left[g_\psi\left(s^t\right)\right], \ t \geq 1 \quad (34)$$

$$\theta_t\left(s^t\right) = \beta E_t \theta_{t+1}\left(s^{t+1}\right)\left[\widetilde{F}_k\left(s^{t+1}\right) + 1 - \delta\right], \ t \geq 0 \quad (35)$$

$$V_c\left(s^0\right) = \theta_0\left(s^0\right) + \Phi A_c \quad (36)$$

$$V_{h^s}\left(s^0\right) = -\theta_0\left(s^0\right)\widetilde{F}_{h^s}\left(s^0\right) + \Phi A_{h^s} \quad (37)$$

$$V_{h^u}\left(s^0\right) = -\theta_0\left(s^0\right)\widetilde{F}_{h^u}\left(s^0\right) + \Phi A_{h^u} \quad (38)$$

$$V_\psi\left(s^0\right) = \theta_0\left(s^0\right)\left[g_\psi\left(s^0\right)\right] + \Phi A_\psi. \quad (39)$$

where $\{\widetilde{F}_{h^s}\left(s^t\right), \widetilde{F}_{h^u}\left(s^t\right), \widetilde{F}_k\left(s^t\right); \forall s^t\}_{t=0}^{\infty}$ are obtained by substituting market clearing conditions (18)-(20) into $\{F_{h^s}\left(s^t\right), F_{h^u}\left(s^t\right), F_k\left(s^t\right); \forall s^t\}_{t=0}^{\infty}$ respectively. The first-order conditions derived in (31)-(39) imply that the system of equations to be solved will be different for $t = 0$ and for $t > 0$. These conditions in a non-stochastic environment are presented in Appendix B.

[10] Note that the multiplier Φ is non-negative and measures the disutility of future tax distortions.

4 Quantitative implementation

In this section we quantitatively solve both the non-stochastic and stochastic optimal policy models. Our solution approach follows Arseneau and Chugh (2012). In particular, we first calibrate the non-stochastic model with exogenous policy. Next, we solve the deterministic Ramsey problem, starting from the exogenous policy steady-state, using non-linear methods. Since we are interested in tax smoothing over the business cycle, we then approximate around the steady-state of the deterministic Ramsey problem to solve the stochastic problem and obtain near steady-state dynamics.

4.1 Functional forms

Following Chari *et al.* (1994) and Stockman (2001), we use a CRRA utility function:

$$u(\cdot) = \frac{\left\{ [c_t(s^t)]^{1-\sigma_1-\sigma_2} [\psi_t(s^t) l_t^s(s^t)]^{\sigma_1} [[1-\psi_t(s^t)] l_t^u(s^t)]^{\sigma_2} \right\}^{\sigma_3}}{\sigma_3} \tag{40}$$

where, σ_1 and σ_2 are the weights to leisure in the utility function and σ_3 is the relative risk aversion parameter.

The production side is given by a CES production function that allows for capital-skill complementarity, since the latter has been shown to match the dynamics of the skill premium in the data (see e.g. Krusell *et al.* (2000), Lindquist (2004), and Pourpourides (2011)):

$$\begin{aligned} F(\cdot) = A_t \{ \mu \left(h_t^{u,f}(s^t) \right)^{\alpha} + & \qquad (41) \\ + (1-\mu) \left[\rho \left(A_t^k k_t^f(s^t) \right)^{\nu} + (1-\rho) \left(h_t^{s,f}(s^t) \right)^{\nu} \right]^{\frac{\alpha}{\nu}} \}^{\frac{1}{\alpha}} & \end{aligned}$$

where, A_t is total factor productivity; A_t^k is the efficiency level of capital equipment; $\alpha < 1$, and $\nu < 1$ are the parameters determining the factor elasticities, i.e. $1/(1-\alpha)$ is the elasticity of substitution between capital and unskilled labour and between skilled and unskilled labour, whereas $1/(1-\nu)$ is the elasticity of substitution between equipment capital and skilled labour; and $0 < \mu, \rho < 1$ are the factor share parameters. In this specification, capital-skill complementarity is obtained if $1/(1-\alpha) > 1/(1-\nu)$.

The above functional form implies that the skill premium, defined as $\frac{w^s(s^t)}{w^u(s^t)}$, can be obtained as:

$$\frac{w^s(s^t)}{w^u(s^t)} = \frac{\widetilde{F}_{h^s}(s^t)}{\widetilde{F}_{h^u}(s^t)} = \frac{(1-\mu)(1-\rho)}{\mu} \frac{[\psi(s^t) h_t^s(s^t)]^{\nu-1}}{\{[1-\psi(s^t)] h_t^u(s^t)\}^{\alpha-1}} (\Xi_t)^{\frac{\alpha}{\nu}-1} \tag{42}$$

where $\Xi_t \equiv \rho \left[\left(A_t^k(s^t) \right)^\nu (k_t(s^{t-1}))^\nu \right] + (1-\rho) \left(\psi(s^t) h_t^s(s^t) \right)^\nu$. The restrictions placed above on the parameters of the production function imply that the skill premium is decreasing in $\psi(s^t)$ and increasing in $k_t(s^{t-1})$, see Appendix C.

The functional form for the relative skill supply is:

$$\widetilde{g}\left[\cdot\right] = \Psi \left[e_t \left(s^t \right) \right]^\gamma \tag{43}$$

where $\Psi > 0$ is the productivity of skill-acquisition; and $0 \leq \gamma < 1$ is the elasticity of the relative skill supply with respect to skill-acquisition expenditure.

Finally, we calculate the effective labour tax rate as the ratio of total tax revenues from both skilled and unskilled sources as a share of total labour income:

$$\tau_t^n\left(s^t\right) = \frac{\tau_t^s\left(s^t\right) w_t^s\left(s^t\right) \psi_t\left(s^t\right) h_t^s\left(s^t\right) + \tau_t^u\left(s^t\right) w_t^u\left(s^t\right)\left(1-\psi_t\left(s^t\right)\right) h_t^u\left(s^t\right)}{w_t^s(s^t)\psi_t(s^t) h_t^s(s^t) + w_t^u(s^t)(1-\psi_t(s^t)) h_t^u(s^t)}. \tag{44}$$

4.2 Exogenous policy and calibration

We next present the calibration and steady-state for the exogenous policy model. In particular, we obtain the steady-state of the following decentralised competitive equilibrium (DCE):

Definition 1. Non-stochastic DCE with exogenous policy

Given initial levels of k_0 and b_0, and the five policy instruments $\{\tau_t^s, \tau_t^u, \tau_t^k, \tau_t^a, g_t^e\}$, the non-stochastic DCE system is characterized by a sequence of allocations $\{c_t, h_t^s, h_t^u, \psi_t, k_{t+1}\}_{t=0}^\infty$, prices $\{w_t^s, w_t^u, r_t, p_t\}_{t=0}^\infty$, and the residual policy instrument $\{b_{t+1}\}_{t=0}^\infty$ such that: (i) households maximise their welfare and firms maximise their profits, taking policy and prices as given; (ii) the government budget constraint is satisfied in each time period and (iii) all markets clear. Thus, imposing the market-clearing conditions (18)-(20), the non-stochastic DCE is comprised of the non-stochastic form of the first-order conditions of the household (6)-(10), the three first-order conditions of the firm (13)-(15), the government budget constraint (16) and the aggregate resource constraint (17).

4.2.1 Calibration

The non-stochastic model with exogenous policy is calibrated so that its steady-state is consistent with the annual US data for 1970-2011.

Utility Table 1 below reports the model's quantitative parameters along with an indication of their source. Starting with the share of leisure for each skill type in utility, σ_1 and σ_2, we calibrate these to 0.35 each so that, in the steady-state, the household devotes about one third of its time to working. The relative risk aversion parameter, $\sigma_3 = -2$ is commonly employed in business cycle models.

Table 1: Model parameters

Parameter	Value	Definition	Source
$0 < \sigma_1 < 1$	0.350	weight to skilled leisure in utility	calibration
$0 < \sigma_2 < 1$	0.350	weight to unskilled leisure in utility	calibration
$\sigma_3 < 0$	-2.000	coefficient of relative risk aversion	assumption
$\frac{1}{1-\alpha} > 0$	1.669	cap. equip. to unskilled labour elasticity	assumption
$0 < \frac{1}{1-\nu} < \frac{1}{1-\alpha}$	0.669	cap. equip. to skilled labour elasticity	assumption
$0 < 1 - \mu < 1$	0.728	share of composite input to output	calibration
$0 < \rho < 1$	0.518	share of cap. equip. to composite input	calibration
$A > 0$	1.000	TFP	assumption
$A^k > 0$	1.000	capital equipment productivity	assumption
$0 \leq \delta \leq 1$	0.007	depreciation rate of capital	calibration
$0 < \beta < 1$	0.960	time discount factor	calibration
$0 \leq \gamma < 1$	0.189	relative skill supply elasticity	calibration
$\Psi > 0$	1.000	productivity of skill-acquisition	assumption
τ^k	0.310	capital income tax rate	data
τ^u	0.200	unskilled labour tax rate	data
τ^s	0.250	skilled labour tax rate	data
τ^n	0.220	effective labour tax rate	data
τ^a	0.000	skill-acquisition expenditure tax rate	assumption
$g^e > 0$	0.047	government spending	calibration

Production The elasticities of substitution between skilled labour and capital and between unskilled labour and capital (or skilled labour) have been estimated by Krusell *et al.* (2000). Following the literature (see e.g. Lindquist (2004), and Pourpourides (2011)), we also use these estimates to set $a = 0.401$ and $\nu = -0.495$. The remaining parameters in the production function are calibrated to ensure the steady-state predictions of the model in asset and labour markets are consistent with the data. More specifically, the labour weight in composite input share $\mu = 0.272$ is calibrated to obtain a labour share of income of approximately equal to 70% and the capital weight in composite input share, $\rho = 0.518$, is calibrated to obtain a skill premium of about 1.64. Both of these targets are consistent with the U.S. data for the period 1970-2011. The target value for the skill premium is from U.S.

Census data and the share of labour income in GDP is from the BEA data on personal income.[11] We also normalize the steady-state values of TFP and capital equipment to unity (i.e. $A = A^k = 1$).

Depreciation and time preference The depreciation rate of capital $\delta = 0.07$ is calibrated to obtain an annual capital to output ratio of about 1.94, which is consistent with the annual data reported by the BEA on capital stocks.[12] The time discount factor, $\beta = 0.96$, is set to obtain a post-tax post-depreciation annual real rate of return on capital of roughly 4.17%, which coheres with the 4.19% obtained in the data from the World Bank.[13]

Relative skill supply To match the share of skilled workers in total population, ψ, of roughly 44% in the data, we set the elasticity of relative skill supply with respect to skill-acquisition, γ, equal to 0.2334. This share is consistent with the data from the 2010 U.S. Census which indicates that 43% of the population has a college degree.[14] It also adheres with a related data set by Acemoglu and Autor (2011) which implies that the average share of the labour force with a college degree is approximately 45%. We normalise skill-acquisition productivity, Ψ to unity.

Tax rates and government spending Finally, we use the ECFIN effective capital and labour tax rates from Martinez-Mongay (2000) to obtain an average tax rate for capital and labour.[15] Therefore, we set the tax rate for capital income $\tau^k = 0.31$ and the two labour income tax rates $\tau^u = 0.20$ and $\tau^s = 0.25$.[16] Given that it is difficult to obtain data which match well with the skill-acquisition expenditure tax rate, τ^a, we set it to zero for the exogenous policy model. We finally set the steady-state value $g^e = 0.0469$,

[11] The data source is the Current Population Survey, 2011 Annual Social and Economic Supplement from the U.S. Census Bureau.

[12] Specifically, the BEA Table 1.1 on fixed-assets has been used to obtain the time series for capital stock for 1970-2011.

[13] The data refers to the annual real interest rate from World Bank Indicators database for the period 1970-2011 (i.e. FR.INR.RINR).

[14] This information is obtained from Table 4 of the Census Bureau, Survey of Income and Program Participation.

[15] In particular, we use the LITR and KITN rates for effective average labour and capital taxes respectively for 1970-2011, as they treat self-employed income as capital income in the calculations.

[16] Note that the calculation of the effective labour income tax rate is equal to 0.22. But since we assume that the skilled and unskilled labour income is taxed differently we decompose the labour income tax into skilled and unskilled tax so as the weighted average of the two tax rates equals 0.22.

to obtain a steady-state debt to output ratio, $b/Y = 53\%$, which is equal to the average debt to GDP ratio obtained in the data.[17]

Steady-state The steady-state of the DCE defined and calibrated above is presented in Table 2. The results indicate that the model's predictions for the great ratios match those implied by the data quite well. For example, in the data for 1970-2011: $\frac{k}{y} = 1.895$, $\frac{c}{y} = 0.640$, $\frac{i}{y} = 0.146$, $\frac{g^e}{y} = 0.203$ and $\frac{b}{y} = 0.530$.[18] Moreover, the share of skill acquisition expenditure in GDP, $\frac{e}{y}$, roughly coheres with US total expenditures for colleges and universities as a share of output equal to 6% for 1970-2010. This data is obtained from the U.S. National Center for Education Statistics, Digest of Education Statistics. As pointed out above, the remaining steady-state variables in the exogenous model, have been calibrated to match their values in the data.

Table 2: Steady-state of exogenous policy

$\frac{c}{y}$	$\frac{k}{y}$	$\frac{i}{y}$	$\frac{e}{y}$	$\frac{b}{y}$	$\frac{g^e}{y}$	$\frac{w^s}{w^u}$	r^{net}	ψ
0.5613	1.9444	0.1361	0.0659	0.5272	0.2367	1.6344	0.0417	0.4400

4.3 Deterministic Ramsey

The deterministic version of the Ramsey problem in (31)-(39) is summarised in Appendix B, (B1-B16) and is solved iteratively, conditional on the calibration described in the previous section. In particular, we first guess a value for Φ and solve equations (B1-B15) for an allocation $\{c_t, h_t^s, h_t^u, \psi_t, k_{t+1}\}_{t=0}^T$. Then we test whether equation (B16) is binding and increase or decrease the value of Φ if the budget is in deficit or surplus respectively.

The initial conditions for the model's state variables are given by the non-stochastic exogenous steady-state (see Table 2). For the terminal values of the forward looking variables, we assume that after T years the dynamic system has converged to its Ramsey steady-state. This implies that the appropriate terminal conditions are obtained by setting the values for these variables equal to those of the preceding period.

The final system is given by $[(15 \times T) + 1]$ equations, which is solved non-linearly using standard numeric methods (see, e.g. Garcia-Milà *et al.* (2010), Adjemian *et al.* (2011), and Angelopoulos *et al.* (2013)). This gives the dynamic transition path from the exogenous to the optimal steady-state. We set $T = 250$ to ensure that convergence is achieved. Our results show

[17]The source of that time series is: FRED Economic Data on Gross Federal Debt as a percentage of GDP, 1970-2011.

[18]Note that if model prediction for the cost of becoming skilled, $\frac{e}{y} = 0.0659$, is added to the $\frac{c}{y}$ ratio from the model, the sum is very close to the $\frac{c}{y}$ ratio in the data.

that this occurs for all endogenous variables within 150 years.[19] After we find the optimal allocation for $\{c_t, h_t^s, h_t^u, \psi_t, k_{t+1}\}_{t=0}^T$ we obtain $w_t^s = \widetilde{F}_{h^s}(t)$, $w_t^u = \widetilde{F}_{h^u}(t)$ and $r_t = \widetilde{F}_k(t)$. Additionally, we solve for τ_t^s, τ_t^u, τ_t^a, τ_t^k and τ_t^n using the non-stochastic form of (6), (7), (8), (10) and (44) respectively.

The Ramsey steady-state is reported in Table 3. The results are consistent with the messages from the literature initiated by Chamley (1986) on dynamic Ramsey taxation in a deterministic environment (see e.g. Ljungqvist and Sargent (2012), ch. 16 for a review of this literature). As expected, allowing the government a complete instrument set results in a zero capital tax rate in the long-run. Compared with the steady-state of exogenous policy, a Ramsey government would increase capital accumulation in the steady-state, by eliminating the *intertemporal* wedge. Moreover, since skilled labour is complementing capital more than unskilled, the Ramsey government would find it optimal to encourage an increase in the relative skill supply, since a higher relative quantity of skilled labour increases the returns to, and thus the accumulation of, physical capital. This is achieved by a small subsidy to skill acquisition expenditure. The fall in the skill premium under Ramsey policy suggests that the increase in the relative skill supply has a relatively stronger quantitative impact than the increase in the capital stock. The Ramsey equilibrium also implies a mild regressivity regarding the long-run labour income taxes, revealing an incentive to encourage the labour supply of skilled hours, consistent with the discussion above. Finally, the government is able to reduce the overall burden of taxation, since it can finance part of the required public spending from accumulated assets. Note that all taxes are reduced compared with the exogenous policy regime.

Table 3: Steady-state of optimal policy

$\frac{c}{y}$	$\frac{k}{y}$	$\frac{i}{y}$	$\frac{e}{y}$	$\frac{b}{y}$	$\frac{g^e}{y}$	$\frac{w^s}{w^u}$
0.5610	2.6428	0.1850	0.0731	-2.4599	0.1809	1.4950

τ^s	τ^u	τ^n	τ^k	τ^a	r^{net}	ψ
0.1188	0.1260	0.1208	0.0000	-0.0353	0.0417	0.4721

We next study the transition dynamics associated with Ramsey policy. Figure 1 illustrates the dynamic paths implied by optimal policy for the capital tax, the two labour taxes, the skill-acquisition expenditure tax and debt to output as the economy evolves from the exogenous steady-state to the Ramsey steady-state. The first panel of Figure 1 shows that in period 1 skilled and unskilled labour are subsidised at rates of 16% and 14.57% respectively; and skill-acquisition expenditure is taxed at a rate of 26.76%. In

[19] See Figure 1 below for an illustration of convergence using the policy instruments.

period 2, skilled and unskilled labour taxes are 15.24% and 14.36% respectively and eventually converge to their steady-state values reported in Table 3. Also in period 2, skill-acquisition is subsidised at a rate of 2.11% and converges to 3.53% in the steady-state. The second panel of Figure 1 shows that in period 1, since capital already in place, capital income is taxed at a confiscatory rate (approximately 210%). In period 2, the capital income tax is 0.27% and then converges slowly to zero. The high capital taxation in the first period allows the government to create a first period stock of assets of approximately the size of GDP, by lending to the household. Government assets increase in future periods and their income is used to subsidise skill-acquisition expenditure and to compensate for the losses from foregone capital income taxation, without the need to resort to high labour income taxes. These transition paths are consistent with previous research.

[Figure 1]

4.4 Stochastic processes

To move to the analysis of the stochastic Ramsey problem, we need to define the stochastic processes that drive economic fluctuations. In what follows we designate a stochastic state s^t at time t that determines exogenous shocks to both the firm's production technologies, (A_t, A_t^k), and to government expenditures (g_t^e). Therefore, the optimal allocation of households will depend on the history of events s^t at time t. Following the literature, A_t, A_t^k and g_t^e are assumed to follow stochastic $AR(1)$ processes:

$$A_{t+1} = (1 - \rho_A)\, A + \rho_A A_t + \varepsilon_{t+1}^{A} \quad (45)$$

$$A_{t+1}^k = (1 - \rho_{A^k})\, A^k + \rho_{A^k} A_t^k + \varepsilon_{t+1}^{A_k} \quad (46)$$

$$g_{t+1}^e = (1 - \rho_{g^e})\, g^e + \rho_{g^e} g_t^e + \varepsilon_{t+1}^{g^e} \quad (47)$$

where ε_t^A, $\varepsilon_t^{A_k}$ and $\varepsilon_t^{g^e}$ are independently and identically distributed Gaussian random variables with zero means and standard deviations given respectively by σ_A, σ_{A^k} and σ_{g^e}.

The values for the $AR(1)$ coefficients and the standard deviations for the government expenditures and capital productivity exogenous processes are data based and are estimated to be: $\rho_{A^k} = 0.90$, $\rho_{g^e} = 0.70$, $\sigma_{A^k} = 0.007$ and $\sigma_g = 0.012$.[20] The autocorrelation parameter of TFP is set equal to 0.95, following Lindquist (2004) and Pourpourides (2011), while σ_A is calibrated to

[20] The government spending series refers to government consumption expenditures and gross investment from NIPA Table 1.1.5 (1970-2011). The capital series refers to productive capital stock and is from the Bureau of Labour Statistics Table 4.1 (1988-2011). Note

match the volatility of output observed in the BEA data.[21] More specifically, the standard deviation for TFP is set $\sigma_A = 0.8\%$ to obtain a volatility for output from 1970-2011 equal to 1.2%.

Table 4: Parameters for stochastic processes

Parameter	Value	Definition	Source
σ_A	0.008	standard deviation of TFP	calibration
ρ_A	0.950	AR(1) coefficient of TFP	data
σ_{A^k}	0.007	standard deviation of capital equipment	data
ρ_{A^k}	0.900	AR(1) coefficient of capital equipment	data
σ_{g^e}	0.012	standard deviation of public spending	data
ρ_{g^e}	0.700	AR(1) coefficient of public spending	data

4.5 Stochastic Ramsey

We next approximate the dynamic equilibrium paths due to three exogenous shocks using first-order accurate decision rules of the equilibrium conditions under optimal policy in (31)-(35), around the optimal deterministic steady-state of these conditions described above.[22] As is common in the literature when characterizing policy dynamics, we also make the auxiliary assumption that the initial state of the economy at $t = 0$ is the steady-state under optimal policy.

As is well known (see e.g. Zhu (1992), Chari *et al.* (1994) and Ljungqvist and Sargent (2012)), the Ramsey problem with state-contingent debt cannot uniquely pin down the capital tax rate. Hence, we follow the literature and calculate the optimal *ex-ante* capital income tax rate (see Appendix D for details):

$$\overline{\tau}^k_{t+1}(s^t) = \frac{\beta E_t u_c(s^{t+1}) \left[\widetilde{F}_k(s^{t+1}) + 1 - \delta\right] - u_c(s^t)}{\beta E_t u_c(s^{t+1}) \widetilde{F}_k(s^{t+1})}. \tag{48}$$

Alternatively, by assuming that government debt is not state-contingent, we can calculate the *ex post* state contingent capital tax (see Appendix E for

that there is no data available prior to 1988 for the productivity of capital. To calculate the statistical properties of the cyclical component of the series, we take logs and apply the HP-filter with smoothing parameter equal to 6.25.

[21] The time series for GDP from 1970-2011 is obtained from NIPA Table 1.1.5. Cyclical output is again calculated using the HP-filter as above.

[22] We use the perturbation methods in Schmitt-Grohé and Uribe (2003) to solve the dynamic model.

the derivation):

$$\widetilde{\tau}_t^k(s^t) = \left(\frac{1}{r_t(s^t)k_t(s^{t-1})}\right)\{g_t(s^t) - \tau_t^a\left(s^t\right) g\left[\psi_t(s^t)\right] - \frac{b_{t+1}\left(s^t\right)}{\overline{R}_t(s^t)} + b_t\left(s^{t-1}\right) -$$
$$- \tau^s(s^t)w_t^s(s^t)\psi_t\left(s^t\right) h_t^s(s^t) - \tau^u(s^t)w_t^u(s^t)\left[1-\psi_t\left(s^t\right)\right] h_t^u(s^t)\} \tag{49}$$

where $\overline{R}_t\left(s_t\right)$ is the state uncontingent or the risk free return to holding government debt. Alternatively, assuming the government employs a state-contingent tax on income from government bonds, we can calculate the private assets tax, $\xi\left(s^{t+1}|s^t\right)$ that applies to taxing jointly the income from assets as (see Appendix E for the derivation):

$$\xi_t\left(s^{t+1}|s^t\right) = \left(\frac{1}{F_k(s^{t+1})k_{t+1}(s^t)+b_{t+1}(s^t)}\right) \times \{g_{t+1}(s^{t+1}) + b_{t+1}\left(s^t\right) - \frac{b_{t+2}\left(s^{t+1}\right)}{\overline{R}_{t+1}(s^{t+1})} -$$
$$-\tau^s(s^{t+1})w_{t+1}^s(s^{t+1})\psi_{t+1}\left(s^{t+1}\right) h_{t+1}^s(s^{t+1}) - \tau^u(s^{t+1})w_{t+1}^u(s^{t+1}) \times$$
$$\times \left[1-\psi_{t+1}\left(s^{t+1}\right)\right] h_{t+1}^u(s^{t+1}) - \tau_{t+1}^a\left(s^{t+1}\right) g\left[\psi_{t+1}(s^{t+1})\right]\}. \tag{50}$$

To calculate the business cycle statistics of the relevant quantities of the model under optimal policy, we conduct simulations by shocking all of the exogenous processes, obtain the required moments for each simulation and then calculate their mean value across the simulations. We undertake 1000 simulations, each 242 periods long and drop the first 200 periods to ensure that the initial conditions do not affect the results. We retain 42 periods in our analysis to match the number of years between 1970 and 2011 used in the calibration.

4.6 Cyclical properties

We next present the results regarding the key second moments of the stochastic optimal policy problem. We conduct this analysis for both the model developed above and the model where the relative skill supply is exogenously determined over the business cycle. This is followed by an impulse response analysis, which allows to investigate the channels through which tax policy works over the business cycle.

4.6.1 Endogenous relative skill supply

We start with the cyclical properties of Ramsey taxation under endogenous relative skill supply. The results on standard deviations and correlations with output, for the endogenous variables of the model as well as the various tax rates that were explained above are summarised in the first three columns of Table 5. The results regarding optimal taxation are largely consistent with the literature and thus extend previous findings to a setup with capital-skill complementarity and endogenous skill supply.

Table 5: Stochastic results

	endogenous ψ			exogenous ψ		
x_i	$\overline{x}_i$	σ_{x_i}	$\rho(x_i, y)$	$\overline{x}_i$	σ_{x_i}	$\rho(x_i, y)$
y	0.2590	0.0229	1	0.2590	0.0211	1
c	0.1452	0.0251	0.9784	0.1452	0.0253	0.9785
k	0.6842	0.0202	0.6070	0.6843	0.0183	0.5958
h^s	0.4090	0.0021	0.4780	0.4090	0.0017	0.2332
h^u	0.2033	0.0093	-0.5381	0.2033	0.0088	-0.5497
ψ	0.4721	0.0034	0.9582	0.4721	0.0000	0.0000
$\frac{w^s}{w^u}$	1.4952	0.0040	-0.9728	1.4948	0.0023	0.4509
τ^s	0.1188	0.0012	0.5158	0.1188	0.0073	-0.9008
τ^u	0.1260	0.0014	-0.2750	0.1261	0.0029	-0.8676
$\psi(1-\tau^s)w^s$	0.2867	0.0257	0.9838	0.2867	0.0255	0.9828
$(1-\psi)(1-\tau^u)w^u$	0.2127	0.0233	0.9894	0.2127	0.0236	0.9889
τ^n	0.1208	0.0009	0.2474	0.1208	0.0061	-0.9046
τ^a	-0.0354	0.0171	0.3053	-0.0354	0.0000	0.0000
$\overline{\tau}^k$	-8.3e-6	0.0004	0.5983	-1.4e-5	0.0004	0.6325
$\widetilde{\tau}^k$	0.0137	0.1291	-0.2087	0.0148	0.1345	-0.2271
ξ	-0.0020	0.0175	0.2092	-0.0022	0.0184	0.2279

In particular, the *ex ante* tax rate on capital is effectively zero and is around zero for every period. Moreover, when debt is not allowed to be state-contingent, the state contingent private assets and *ex post* capital taxes are near zero, have low correlations with output and are the most volatile of the tax instruments. These results are similar to findings in the literature to date. Also consistent with the labour tax-smoothing results in the literature, both labour taxes have very low standard deviations relative to output, as the government finds it optimal to minimise the distortions introduced by labour taxes over the business cycle by keeping them relatively smooth and letting the remaining state-contingent policy instruments respond to exogenous shocks. However, they exhibit different correlations with output. The tax rate on skilled labour income is pro-cyclical, whereas the tax rate on unskilled labour income is mildly counter-cyclical. The skill-acquisition tax is the least smooth of the tax instruments when debt is state-contingent and fluctuates nearly as much as output. Moreover, it is mildly pro-cyclical.

Finally, the labour income taxes and the *ex ante* capital income tax in this model inherit the properties of the exogenous processes. As can be seen in Table 6, the autocorrelations of these instruments follow the autocorrelations of the exogenous processes, so that when shocks are autocorrelated as in Table 4, so are the tax rates. However, if we assume that the shocks follow *iid* processes, the autocorrelation of the tax rates generally becomes very small. On the contrary, the autocorrelations of the *ex post* capital tax and of the private assets tax do not follow the autocorrelations of the exogenous processes. This is again similar to previous findings.

Table 6: Autocorrelations

	autocorrelated shocks		*iid*shocks	
	endogenous ψ	exogenous ψ	endogenous ψ	exogenous ψ
τ^s	0.7714	0.9101	-0.0431	-0.0262
τ^u	0.9116	0.9403	-0.0487	0.3212
τ^n	0.7660	0.9182	-0.0637	-0.0026
τ^a	0.8304	1.0000	0.0570	1.0000
$\overline{\tau}^k$	0.7402	0.7429	-0.0435	-0.0398
$\widetilde{\tau}^k$	-0.1602	-0.1596	-0.5009	-0.4985
ξ	-0.1731	-0.1711	-0.5016	-0.4982

4.6.2 Exogenous relative skill supply

We next examine how the prescriptions for optimal policy are affected by a friction in the labour market that does not permit changes in the relative skill supply over the business cycle. As discussed in the introduction, this restriction is empirically relevant.[23] To analyse the effects of a fixed relative skill supply over the business cycle, we obtain the first-order conditions for optimal policy incorporating this rigidity and then approximate these conditions around the Ramsey deterministic steady-state with endogenous $\psi_t(s^t)$ in Table 3. The latter avoids approximating around the steady-state in which the relative skill supply is restricted over both the short- and long-run. Thus, we set $\psi_t(s^t)$, for each possible history s^t, to be equal to the steady-state value from the deterministic Ramsey problem with endogenous $\psi_t(s^t)$ in Table 3. This also means that skill-acquisition expenditure $e_t(s^t)$ and the respective tax rate $\tau_t^a(s^t)$ are also set to their respective values in Table 3.[24] The results pertaining to the business cycle properties of the econ-

[23]Note that the model with an endogenously chosen relative skill supply does not capture this feature. In particular, when the model is simulated under the exogenous processes in Section 4.4, it produces an HP filtered series for $\psi_t(s^t)$, which has a correlation with similarly detrended output of about 60% and a relative-to-output standard deviation of around 50%.

[24]Note we keep $\tau_t^a(s^t)$ constant when skill-acquisition expenditure remains constant, since there is no margin in the household decision for $\tau_t^a(s^t)$ to affect. Hence, it is equivalent to a lump-sum tax, the optimal choice of which is ruled out in Ramsey second-best

omy under optimal policy in this case are presented in the last three columns of Table 5.

These results first suggest that the properties of asset taxation do not change. However, there are important differences regarding labour income taxation. In particular, the two labour income taxes become quantitatively more volatile, so that the effective labour income tax rate, $\tau_t^n(s^t)$, is about seven times more volatile. Also note that the labour tax volatility increases asymmetrically, so that $\tau_t^s(s^t)$ is about two-and-a-half times more volatile than $\tau_t^u(s^t)$. Finally, both labour taxes become strongly counter-cyclical. Thus, under capital-skill complementarity, imposing the restriction that the relative skill supply does not change over the business cycle has important implications for the business cycle properties of labour income taxation.

4.6.3 Labour market wedges

The key to understanding these changes is to note that the rigidity of $\psi_t(s^t)$ over the business cycle creates a distortion in the labour markets that is reflected in the difference between the *atemporal* first-order conditions of the household given by (6)-(7) and the corresponding conditions when the relative skill supply is fixed, $\overline{\psi}$. This distortion drives a wedge between the average net returns to labour supply in the perfect and imperfect labour markets, or, alternatively, a wedge between the marginal rates of substitution between leisure and consumption in the perfect and imperfect labour markets. Thus these wedges for skilled and unskilled workers respectively can be defined as follows:

$$\begin{aligned} lw_t^s(s^t) &= \left[1-\tau_t^s(s^t)\right]\psi_t(s^t)\,w_t^s(s^t) - \left[1-\widehat{\tau}_t^s(s^t)\right]\overline{\psi}\widehat{w}_t^s(s^t) \\ lw_t^u(s^t) &= \left[1-\tau_t^u(s^t)\right]\left[1-\psi_t(s^t)\right]w_t^u(s^t) - \left[1-\widehat{\tau}_t^u(s^t)\right]\left(1-\overline{\psi}\right)\widehat{w}_t^u(s^t) \end{aligned} \tag{51}$$

where hatted variables denote the case when the relative skill supply is fixed.

Given that in both models considered here (i.e. with flexible and rigid relative skill supply) it is assumed that the government needs to resort to distortionary taxation so that the first-best cannot be achieved in either case. Hence, the best that the government can do is to achieve the second-best allocations in the labour markets represented by Ramsey taxation under flexible relative skill supply in (6) and (7). This is reflected in the definition of the labour wedges created by the rigidity in the relative skill supply in (51).

Our results make clear that the government wishes to minimise these wedges over the business cycle and this is achieved by setting $\widehat{\tau}_t^s(s^t)$ and

analysis.

$\widehat{\tau}_t^u(s^t)$ so that paths for the skilled and unskilled average net return under the market distortion are as close as possible to the paths of the corresponding quantities without the market distortion. Table 5 clearly shows that second moments of these returns are very similar and this is further confirmed when we examine the impulse responses below. In contrast, the *intertemporal* margins are not directly affected by the rigidity in the relative skill supply. Hence, the optimal policies regarding asset taxation are not qualitatively different between the two models. Finally, the results relating to the autocorrelation properties of the instruments generally follow the same pattern as when the relative skill supply is endogenous. The natural exception here is for the autocorrelation properties of τ^a, which is constant when ψ is exogenous since skill acquisition expenditure is constant. Thus τ^a has a unit AR(1) parameter for both the autocorrelated and *iid* cases.

4.6.4 Impulse responses

To further explain the previous results and examine the optimal response of taxation to changes in exogenous productivity and government spending, we plot the impulse responses of key endogenous variables after a temporary 1% shock to the exogenous distributions in ε_t^A, $\varepsilon_t^{A_k}$ and $\varepsilon_t^{g^e}$. These plots are shown in Figures 2-4 below.

[Figures 2-4]

After a positive TFP or capital equipment shock (see Figures 2 and 3), the capital stock, k_t, increases, since the productivity of capital increases. As shown earlier (see Appendix C), this tends to increase the returns to skilled hours more than the return to unskilled, given capital-skill complementarity. In the flexible labour markets model, the increase in the returns to skilled labour also leads to an increase in the relative supply of skill, ψ_t, which, other things equal, tends to decrease the skill premium (see Appendix C). These two forces, on balance, lead to a fall in the skill premium, $\frac{w_t^s}{w_t^u}$, shown in the Figures. The government finds it optimal to respond to these shocks by keeping the labour income taxes (τ_t^s and τ_t^u) relatively smooth, consistent with the tax smoothing literature. Optimal policy also encourages the accumulation of skill by decreasing τ_t^a.[25]

Under the relative skill supply restriction, the increase in the capital stock cannot be followed by an increase in ψ_t (see again Figures 2 and 3). Therefore,

[25] However, note that the smoothness of the labour income taxes is not due to the skill-acquisition subsidy, since a version of the model where $\tau_t^a(s^t)$ is fixed over the business cycle provides very similar second moments and impulse responses. These results are not presented here to save on space but are available on request.

the returns to skilled and unskilled labour, w_t^s and w_t^u, respectively, now follow different paths, summarised by the increase in the skill premium. *Ceteris paribus*, this drives a wedge between the average net returns to skilled and unskilled labour hours under the restricted model, relative to those from the flexible labour markets model. To minimise the effects of the relative labour supply distortion, the government adjusts the optimal response of the labour income taxes, as can be seen in the plots for these returns. It achieves this by by keeping the marginal rates of substitution between leisure and consumption for the two types of labour at roughly the same levels as under a fully flexible labour market. Indeed, the response becomes more counter-cyclical, to smooth the response of average net returns to skilled and unskilled labour, so that these last two quantities exhibit, post shock, effectively identical responses with their corresponding quantities in the flexible labour market. Note also that the change in τ_t^s is larger than τ_t^u, since, given capital-skill complementarity, w_t^s is affected more by the increase in the capital stock than w_t^u. Thus a larger adjustment in policy is required.

A temporary reduction in government spending in Figure 4, does not have direct productivity effects in these models. However, it allows the government to briefly reduce the tax burden on labour income and thus encourage labour supply. In the model with endogenous relative skill supply, a small reduction in τ_t^s increases the average net return to skilled labour both directly and indirectly, via the induced increase in ψ_t. The latter happens because the increase in skilled labour raises the return to capital as well and thus the returns to investing into skill-acquisition. On the contrary, under the restricted relative skill supply assumption, the indirect effect is missing and thus τ_t^s needs to be increased by more, to maintain the same average net return to skilled labour hours. The unskilled labour supply does not affect capital accumulation as much (given capital-skill complementarity). Hence it does not need to be changed by as much under endogenous relative skill supply. In turn, this implies that no big changes are required in the optimal response to τ_t^u when relative skill supply is fixed, to maintain the same average net return to unskilled labour hours.

5 Conclusions

Motivated by the empirical relevance of the wage-skill premium and the roles played by capital-skill complementarity, the relative supply of skilled labour and capital augmenting technical change, this paper contributed to the tax smoothing literature by undertaking a normative investigation of the quantitative properties of optimal taxation of capital and labour income, as well as skill-acquisition expenditure, in the presence of aggregate shocks to total factor productivity (TFP), capital equipment productivity and government spending.

Our main finding was that under capital-skill complementarity, a friction that did not allow the relative supply of skill to adjust in response to aggregate shocks, significantly changed the cyclical properties of optimal labour taxes. In particular, we first showed that under endogenous relative skill supply, the optimal labour taxes for both skilled and unskilled labour income were smooth, with the volatility of the skilled income tax being marginally lower. We also found that the skilled tax moves pro-cyclically with output and the unskilled tax was mildly counter-cyclical. These results were largely consistent with the literature and extended previous findings to a setup with capital-skill complementarity and endogenous skill supply.

We also found that, when the relative skill supply was constrained to remain constant over the business cycle, the prescriptions for optimal policy markedly changed. In particular, we found that the volatility of taxes increased significantly, so that the standard deviation of the effective average labour income tax was about seven times higher than the perfect labour markets case, while the volatility of the skilled labour income tax was about two-and-a-half times higher than that of the unskilled labour income tax. Moreover, both taxes became strongly counter-cyclical. We further demonstrated that the key to explaining these changes was that the government found it optimal to adjust labour income tax rates to alter the average net returns to skilled and unskilled labour hours, so that their dynamic behaviour under restricted skill supply is the same as under flexible skill supply.

Our results additionally revealed that the skill heterogeneity considered, irrespective of the presence of the labour market friction, did not affect the results obtained in the literature regarding the cyclical behaviour of asset taxes. In particular, the *ex ante* tax rate on capital was around zero for every period, the state contingent private assets and *ex post* capital taxes were near zero and are the most volatile of the tax instruments. We also found that the skill-acquisition tax was the least smooth of the tax instruments when debt was state-contingent and fluctuated nearly as much as output. Finally, irrespective of the model variant examined, all of the policy instruments, except the *ex post* capital tax and the private assets tax inherited the persistence properties of the shocks.

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Appendix A: Household's first-order conditions

The household's first-order conditions for consumption, skilled labour supply, unskilled labour supply, debt, capital and the relative skill supply are given respectively by the following relations:

$$u_c(s^t) = \lambda_t\left(s^t\right) \tag{A1}$$

$$u_{h^s}(s^t) = -\lambda_t(s^t)\psi_t(s^t)\left\{\left[1-\tau_t^s(s^t)\right]w_t^s\left(s^t\right)\right\} \tag{A2}$$

$$u_{h^u}(s^t) = -\lambda_t(s^t)\left[1-\psi_t(s^t)\right]\left[1-\tau_t^u(s^t)\right]w_t^u\left(s^t\right) \tag{A3}$$

$$\pi_t\left(s^t\right)\lambda_t(s^t)p_t\left(s_{t+1}\mid s^t\right) = \beta\pi_{t+1}\left(s^{t+1}\right)\lambda_{t+1}(s^{t+1}) \tag{A4}$$

$$\begin{aligned}\pi_t\left(s^t\right)\lambda_t(s^t) = \beta\sum_{s_{t+1}}\{\pi_{t+1}\left(s^{t+1}\right)\lambda_{t+1}(s^{t+1})\times \\ \times\left[r_{t+1}\left(s^{t+1}\right)\left[1-\tau_{t+1}^k\left(s^{t+1}\right)\right]+(1-\delta)\right]\}\end{aligned} \tag{A5}$$

$$\begin{aligned}u_\psi(s^t) = -\lambda_t(s^t)\{h_t^s\left(s^t\right)\left[1-\tau_t^s\left(s^t\right)\right]w_t^s\left(s^t\right) - h_t^u\left(s^t\right)\times \\ \times\left[1-\tau_t^u\left(s^t\right)\right]w_t^u\left(s^t\right) - \left[1+\tau_t^a\left(s^t\right)\right]\left[g_\psi\left(s^t\right)\right]\}.\end{aligned} \tag{A6}$$

Appendix B: Deterministic Ramsey system

In a non-stochastic environment, the first-order conditions derived in (31)-(39) of the main text become:

- for $t=0$:

$$V_{h^s}(0) = -\left[V_c(0)-\Phi A_c\right]\widetilde{F}_{h^s}(0)+\Phi A_{h^s} \tag{B1}$$

$$V_{h^u}(0) = -\left[V_c(0)-\Phi A_c\right]\widetilde{F}_{h^u}(0)+\Phi A_{h^u} \tag{B2}$$

$$V_\psi(0) = \left[V_c(0)-\Phi A_c\right]\left[g_\psi(0)\right]+\Phi A_\psi \tag{B3}$$

$$V_c(0) = \beta V_c(1)\left[\widetilde{F}_k(1)+1-\delta\right]+\Phi A_c \tag{B4}$$

$$\widetilde{F}\left[\cdot(0)\right] = c_0+g^e+g\left(\psi_0\right)+k_1-(1-\delta)k_0 \tag{B5}$$

- for $t = 1, 2, 3...T-1$:

$$V_{h^s}(t) = -V_c(t)\,\widetilde{F}_{h^s}(t) \tag{B6}$$

$$V_{h^u}(t) = -V_c(t)\,\widetilde{F}_{h^u}(t) \tag{B7}$$

$$V_{\psi}(t) = V_c(t)\left[g_{\psi}(t)\right] \tag{B8}$$

$$V_c(t) = \beta V_c(t+1)\left[\widetilde{F}_k(t+1) + 1 - \delta\right] \tag{B9}$$

$$\widetilde{F}\left[\cdot(t)\right] = c_t + g^e + g(\psi_t) + k_{t+1} - (1-\delta)k_t \tag{B10}$$

- for $t = T$:

$$V_{h^s}(T) = -V_c(T)\,\widetilde{F}_{h^s}(T) \tag{B11}$$

$$V_{h^u}(T) = -V_c(T)\,\widetilde{F}_{h^u}(T) \tag{B12}$$

$$V_{\psi}(T) = V_c(T)\left[g_{\psi}(T)\right] \tag{B13}$$

$$1 = \beta\left[\widetilde{F}_k(T) + 1 - \delta\right] \tag{B14}$$

$$\widetilde{F}\left[\cdot(T)\right] = c_T + g^e + g(\psi_T) + k_{T+1} - (1-\delta)k_T \tag{B15}$$

- lifetime implementability constraint:

$$\sum_{t=0}^{T}\beta^t[u_c(t)\,c_t + u_{h^s}h_t^s + u_{h^u}h_t^u + \Omega_t] - A = 0 \tag{B16}$$

where $A = u_c(0)\left\{b_0 + \left[(1-\tau_0^k)\widetilde{F}_k(0) + (1-\delta)\right]k_0\right\}$, the Lagrange multiplier θ_t has been replaced with $V_c(t)$ using (31) and (36) in the main text and the notation $X(t)$ denotes the time period t quantity of X.

Appendix C: The effects of k_t and ψ_t on the skill premium

Differentiating the skill premium, given by (42) in the main text,with respect to k_t we have (note that we do not use the s^t notation to keep the presentation more parsimonious):

$$\frac{\partial\left(\frac{w_t^s}{w_t^u}\right)}{\partial k_t} = \{A_t^k\nu\rho\left(A_t^k k_t\right)^{\nu-1}(\psi_t h_t^s)^{\nu-1}\left[(1-\rho)(\psi_t h_t^s)^{\nu} + \rho\left(A_t^k k_t\right)^{\nu}\right]^{\frac{a}{\nu}-2}\times$$
$$\times\left((1-\psi_t)\,h_t^u\right)^{1-\alpha}(1-\mu)(1-\rho)\left(\frac{a}{\nu}-1\right)\}\div\mu$$

This is positive if $a > \nu, a, \nu < 1, 0 < \rho, \mu < 1$.

Differentiating (42) with respect to ψ_t gives:

$$\frac{\partial\left(\frac{w_t^s}{w_t^u}\right)}{\partial\psi_t} = -h_t^u\left(\psi_t h_t^s\right)^\nu(1-\mu)(1-\rho)\left[(1-\rho)\left(\psi_t h_t^s\right)^\nu + \rho\left(A_t^k k_t\right)^\nu\right]^{\frac{a}{\nu}-2}\times$$
$$\times\left[\begin{array}{c}\left(\psi_t h_t^s\right)^\nu\left[(1-\rho)(1-a)\right]+\\ +\rho\left(A_t^k k_t\right)^\nu\left[(1-\nu)+\psi_t(\nu-a)\right]\end{array}\right]\div$$
$$\div\left[\mu h_t^s\psi_t^2\left((1-\psi_t)h_t^u\right)^\alpha\right]$$

This expression is negative if $[(1-\nu)+\psi_t(\nu-a)] > 0$ or $\frac{1-\nu}{\alpha-\nu} > \psi_t$, which is true because $\frac{1-\nu}{\alpha-\nu} > 1$, since $1 > \alpha \Rightarrow 1-\nu > \alpha-\nu$ and $0 < \psi_t < 1$.

Appendix D: Ex ante capital tax

Assume that the government uses a capital tax that is not state-contingent, so that its value for period $t+1$ is decided using the history s^t. Define this uncontingent tax as $\overline{\tau}_{t+1}^k(s^t)$ and note that it needs to satisfy the Euler-equation from (10) in the main text, so that the Ramsey allocations are preserved:

$$u_c(s^t) = \beta E_t\left\{u_c(s^{t+1})\left[\widetilde{F}_k(s^{t+1})\left[1-\tau_{t+1}^k(s^{t+1})\right]+1-\delta\right]\right\} \tag{C1}$$

where we have used $\widetilde{F}_k(s^{t+1}) = r_{t+1}(s^{t+1})$. Hence, $\overline{\tau}_{t+1}^k(s^t)$ needs to satisfy:

$$u_c(s^t) = \beta E_t\left\{u_c(s^{t+1})\left[\widetilde{F}_k(s^{t+1})\left[1-\overline{\tau}_{t+1}^k(s^t)\right]+1-\delta\right]\right\}. \tag{C2}$$

By comparing (C2) with (C1), we see that $\overline{\tau}_{t+1}^k(s^t)$ needs to satisfy:

$$E_t\left\{u_c(s^{t+1})\left[\widetilde{F}_k(s^{t+1})\left[1-\overline{\tau}_{t+1}^k(s^t)\right]+1-\delta\right]\right\} = \tag{C3}$$
$$= E_t\left\{u_c(s^{t+1})\left[\widetilde{F}_k(s^{t+1})\left[1-\tau_{t+1}^k(s^{t+1})\right]+1-\delta\right]\right\}$$

implying that:

$$\overline{\tau}_{t+1}^k(s^t) = \frac{E_t u_c(s^{t+1})\left[\tau_{t+1}^k(s^{t+1})\widetilde{F}_k(s^{t+1})\right]}{E_t u_c(s^{t+1})\widetilde{F}_k(s^{t+1})}. \tag{C4}$$

This gives $\overline{\tau}_{t+1}^k(s^t)$ the *ex ante* capital tax interpretation, since, by multiplying both numerator and denominator in (C4) by $k_{t+1}(s^t)$, this expression provides the expected tax revenue from capital income as share of the expected capital income, where the expectation is calculated using information at period t.

To obtain the *ex ante* rate stated in equation (48) of the main text, we first expand the Euler-equation (C1):

$$u_c(s^t) = \beta E_t u_c(s^{t+1})\widetilde{F}_k(s^{t+1}) - \beta E_t u_c(s^{t+1})\tau^k_{t+1}(s^{t+1})\widetilde{F}_k(s^{t+1}) + \\ + \beta E_t u_c(s^{t+1})\left(1-\delta\right) \tag{C5}$$

and note that $E_t u_c(s^{t+1})\tau^k_{t+1}(s^{t+1})\widetilde{F}_k(s^{t+1})$ in (C5) equals $\overline{\tau}^k_{t+1}(s^t)E_t u_c(s^{t+1})\times$ $\times\widetilde{F}_k(s^{t+1})$, using (C4). Substituting this expression back into (C5) we obtain:

$$u_c(s^t) = \beta E_t u_c(s^{t+1})\widetilde{F}_k(s^{t+1}) - \beta\overline{\tau}^k_{t+1}(s^t)E_t u_c(s^{t+1})\widetilde{F}_k(s^{t+1}) + \\ + \beta E_t u_c(s^{t+1})\left(1-\delta\right). \tag{C6}$$

Finally solving (C6) for $\overline{\tau}^k_{t+1}(s^t)$ gives the *ex ante* capital tax rate reported in equation (48) of the main text.

Appendix E: Uncontingent debt

Ex-post capital tax

The treatment of state-uncontingent debt and presentation follows Chari *et al.* (1994) and Ljungqvist and Sargent (2012, ch. 16). Assume that the government issues uncontingent debt, $b_{t+1}\left(s^t\right)$ which has a risk-free return $\overline{R}_t\left(s^t\right)$. The budget constraint of the government in period t is written as:

$$g_t(s^t) = \tau^s(s^t)w^s_t(s^t)\psi_t\left(s^t\right)h^s_t(s^t) + \tau^u(s^t)w^u_t(s^t)\left[1-\psi_t\left(s^t\right)\right]h^u_t(s^t) + \\ + \tau^a_t\left(s^t\right)g\left[\psi_t(s^t)\right] + \tau^k_t(s^t)r_t(s^t)k_t(s^{t-1}) + \frac{b_{t+1}\left(s^t\right)}{\overline{R}_t(s^t)} - b_t\left(s^{t-1}\right). \tag{D1}$$

The budget constraint of the household in period t is given by:

$$c_t\left(s^t\right) + k_{t+1}\left(s^t\right) + \frac{b_{t+1}\left(s^t\right)}{\overline{R}_t(s^t)} + \left[1+\tau^a_t\left(s^t\right)\right]g\left[\psi_t(s^t)\right] = \left(1-\tau^s_t\left(s^t\right)\right)\times \\ \times w^s_t\left(s^t\right)\psi_t\left(s^t\right)h^s_t\left(s^t\right) + \left[1-\tau^u_t\left(s^t\right)\right]w^u_t\left(s^t\right)\left[1-\psi_t\left(s^t\right)\right]h^u_t\left(s^t\right) + \\ + \left(1-\delta\right)k_t\left(s^{t-1}\right) + \left[1-\tau^k_t\left(s^t\right)\right]r_t\left(s^t\right)k_t\left(s^{t-1}\right) + b_t\left(s^{t-1}\right) \tag{D2}$$

which implies that the first-order condition with respect to holding bonds is given by:

$$\frac{1}{\overline{R}_t\left(s^t\right)} = \beta E_t\frac{u_c(s^{t+1})}{u_c(s^t)}. \tag{D3}$$

Note that the right-hand side of (A4) needs to be the same as the right-hand side of the first-order condition with respect to bonds in the case of state-contingent debt, so that the implied allocations from the two problems (i.e. with and without state-contingent debt) are the same. In turn, this implies that the risk-free (or uncontingent) return needs to satisfy:

$$\frac{1}{\overline{R}_t\left(s^t\right)} = \sum_{s^{t+1}|s^t} p_t\left(s_{t+1} \mid s^t\right). \tag{D4}$$

To obtain an expression for $b_{r+1}\left(s^r\right)$ for a given period r, we work as follows. We multiply the budget constraint of the household in (D2) for periods r and $r+1$ by $\pi_r\left(s^r\right)$ and $\pi_{r+1}\left(s^{r+1}\right)$ respectively, sum the resulting budget constraint in $r+1$ over all possible realisations s_{r+1} and add it to the budget constraint in period r. We then use the first-order conditions of the household to simplify the expression and continue this forward iterative process until time period $T \to \infty$. By imposing the appropriate transversality conditions we obtain an expression for $b_{r+1}\left(s^r\right)$ as a function of identified equilibrium paths given in:

$$\begin{aligned} b_{r+1}\left(s^r\right) = &\bar{R}_r\left(s^r\right) \sum_{t=r+1}^{\infty} \sum_{s^t} \frac{\beta^{t-r}\pi_t\left(s^t\right)\left[u_c(s^t)c_t\left(s^t\right)+u_{h^s}(s^t)h_t^s\left(s^t\right)+u_{h^u}(s^t)h_t^u\left(s^t\right)+\Omega_t\left(s^t\right)\right]}{\pi_r(s^r)u_c(s^r)} - \\ &-\bar{R}_r\left(s^r\right)k_{r+1}\left(s^r\right) \end{aligned} \tag{D5}$$

where, $\Omega_t\left(s^t\right)$ is defined in the main text under equation (28). Hence we can use (D3) to obtain $\overline{R}_t\left(s^t\right)$, (D5) to find $b_{t+1}\left(s^t\right)$ and finally (D1) to calculate the *ex-post* capital tax reported in equation (49) of the main text.

Private assets tax

Assume that the government issues uncontingent debt, $b_{t+1}\left(s^t\right)$, which has a risk-free return $\overline{R}_t\left(s^t\right)$, satisfying (D4), but which is taxed using a state-contingent tax $\upsilon_{t+1}\left(s^{t+1}\right)$. The budget constraint of the government is now written as:

$$\begin{aligned} g_t(s^t) = &\tau^s(s^t)w_t^s(s^t)\psi_t\left(s^t\right)h_t^s(s^t) + \tau^u(s^t)w_t^u(s^t)\left[1-\psi_t\left(s^t\right)\right]h_t^u(s^t)+ \\ &+\tau_t^a\left(s^t\right)g\left[\psi_t(s^t)\right] + \tau_t^k(s^t)r_t(s^t)k_t(s^{t-1})+ \\ &+\frac{b_{t+1}\left(s^t\right)}{\overline{R}_t\left(s^t\right)} - \left[1-\upsilon_t\left(s^t\right)\right]b_t\left(s^{t-1}\right) \end{aligned} \tag{D6}$$

while the budget constraint of the household becomes:

$$\begin{aligned}&c_t\left(s^t\right)+k_{t+1}\left(s^t\right)+\tfrac{b_{t+1}\left(s^t\right)}{\overline{R}_t\left(s^t\right)}+\left[1+\tau_t^a\left(s^t\right)\right]g\left[\psi_t(s^t)\right]=\\&=\left[1-\tau_t^s\left(s^t\right)\right]w_t^s\left(s^t\right)\psi_t\left(s^t\right)h_t^s\left(s^t\right)+\left[1-\tau_t^u\left(s^t\right)\right]w_t^u\left(s^t\right)\times\\&\times\left[1-\psi_t\left(s^t\right)\right]h_t^u\left(s^t\right)+\left(1-\delta\right)k_t\left(s^{t-1}\right)+\left[1-\tau_t^k\left(s^t\right)\right]r_t\left(s^t\right)\times\\&\times k_t\left(s^{t-1}\right)+\left[1-\upsilon_t\left(s^t\right)\right]b_t\left(s^{t-1}\right)\end{aligned}\qquad\text{(D7)}$$

which implies that the first-order condition with respect to holding bonds becomes:

$$\frac{1}{\overline{R}_t\left(s^t\right)}=\sum_{s^{t+1}|s^t}\beta\pi_{t+1}\left(s^{t+1}\mid s^t\right)\frac{u_c(s^{t+1})}{u_c(s^t)}\left[1-\upsilon_{t+1}\left(s^{t+1}\right)\right].\qquad\text{(D8)}$$

The introduction of the new assets tax has to be such that the equilibrium allocations obtained without it are respected. Hence the asset tax must be such that makes the right-hand side of (A4) and (D8) equal. Hence, the asset tax must satisfy:

$$E_t u_c(s^{t+1})\upsilon_{t+1}\left(s^{t+1}\right)=0\qquad\text{(D9)}$$

which implies that at time period t, the expected value of the asset tax in period $t+1$, valued in terms of utility, has to be equal to zero. Therefore, (D9) implies that $\overline{R}_t\left(s^t\right)$ in this case is given by (D3) as well. Moreover, to obtain (D5), we substitute household budget constraints in (D7) forward, using the household first-order conditions, the transversality conditions, the restriction in (D9) and the restriction that the asset tax in the initial period under consideration is zero. Note that this restriction is equivalent to making the zero capital tax assumption in the initial period.

The private assets tax is defined as the tax revenue from assets over income from assets. In particular:

$$\xi_t\left(s^{t+1}|s^t\right)=\frac{\tau_{t+1}^k(s^{t+1})F_k(s^{t+1})k_{t+1}(s^t)+\upsilon_{t+1}\left(s^{t+1}\right)b_{t+1}\left(s^t\right)}{F_k(s^{t+1})k_{t+1}(s^t)+b_{t+1}\left(s^t\right)}.\qquad\text{(D10)}$$

Solving (D6) for $\upsilon_t\left(s^t\right)b_t\left(s^{t-1}\right)$ and substituting this into (D10) we have the expression for $\xi_t\left(s^{t+1}|s^t\right)$ reported in equation (50) of the main text.

Figure 1: Transition paths of the policy instruments

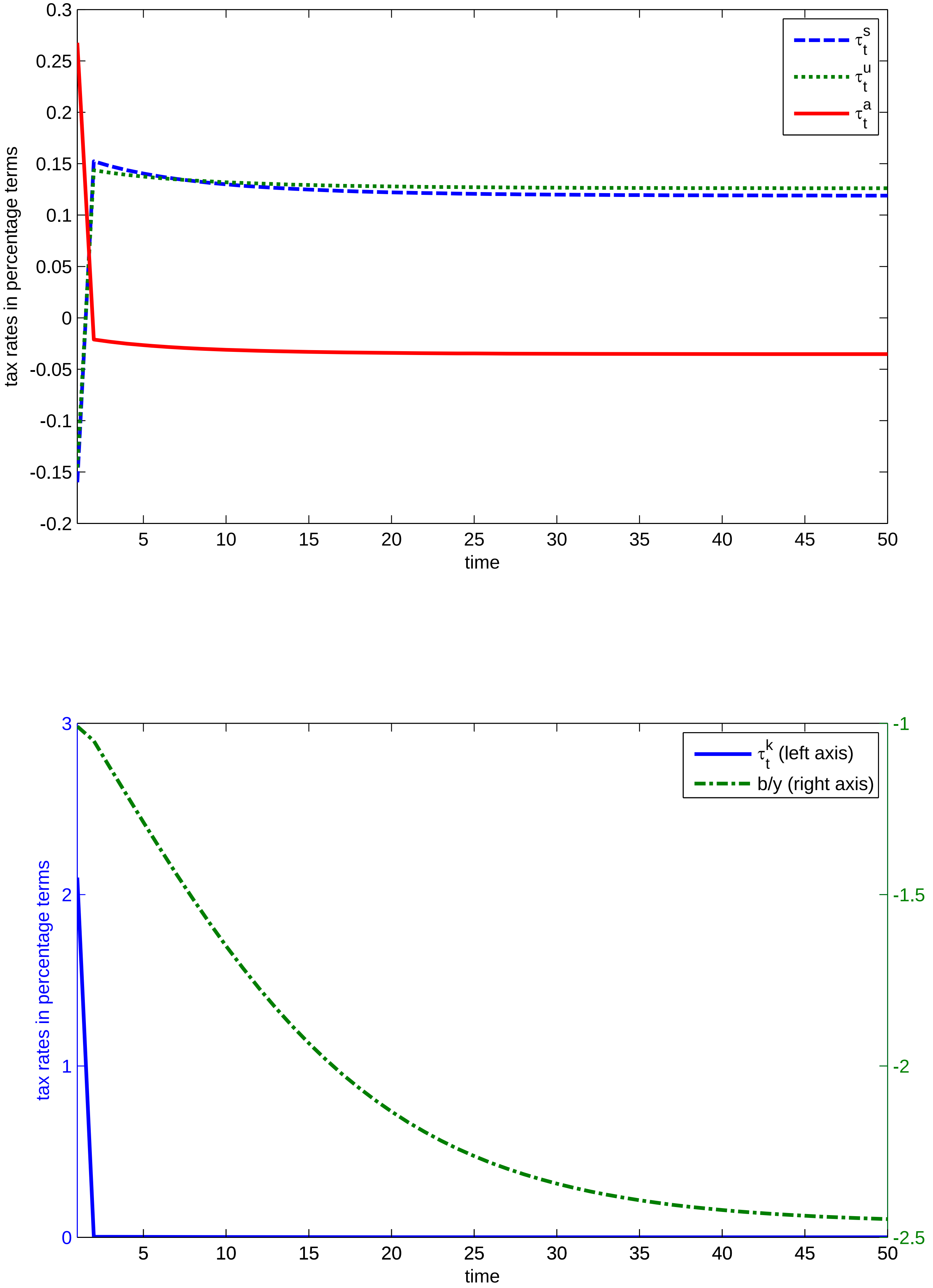

Figure 2: Impulse responses to 1% temporary shock to TFP

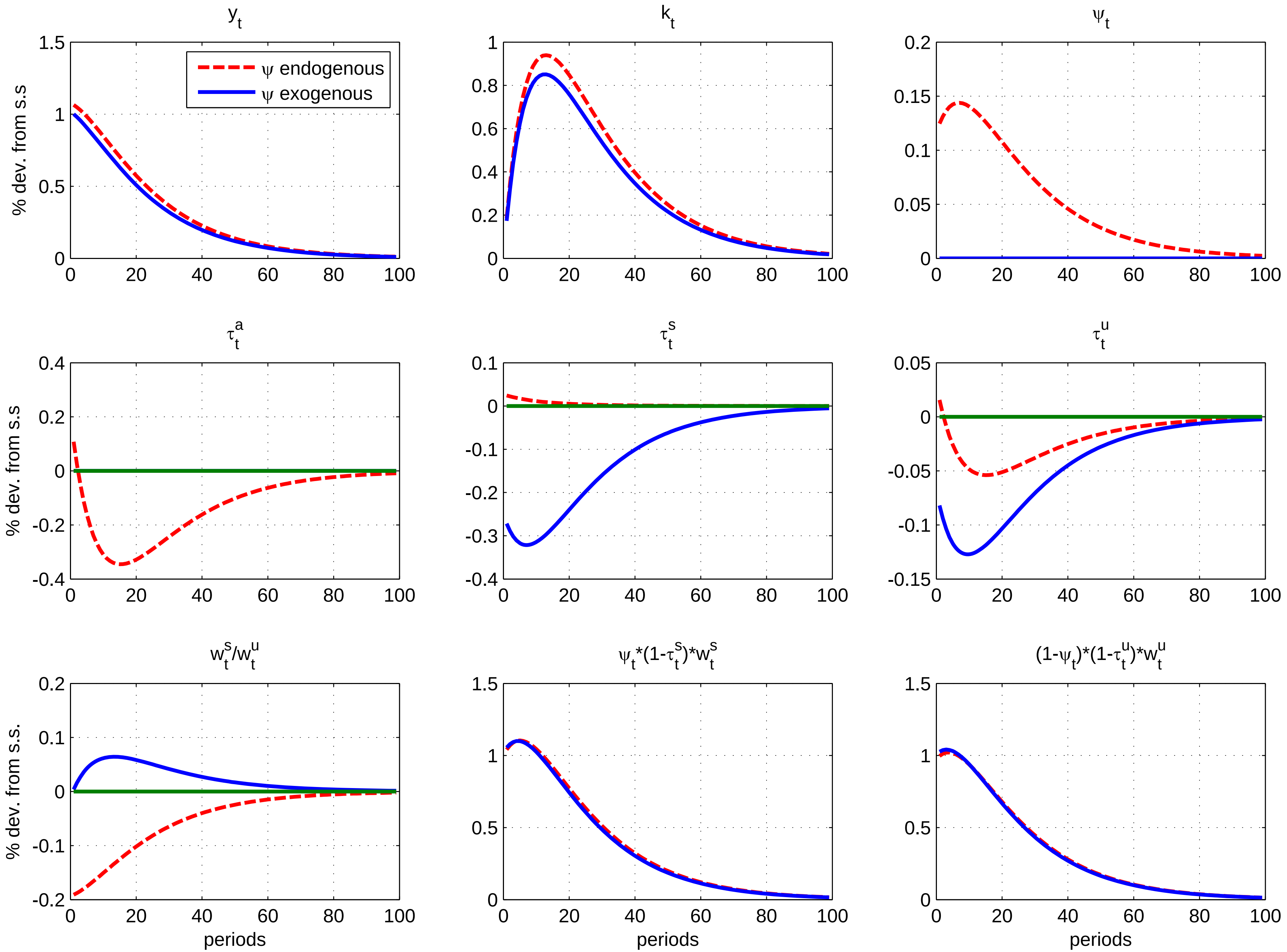

Figure 3: Impulse responses to 1% temporary shock to capital equipment productivity

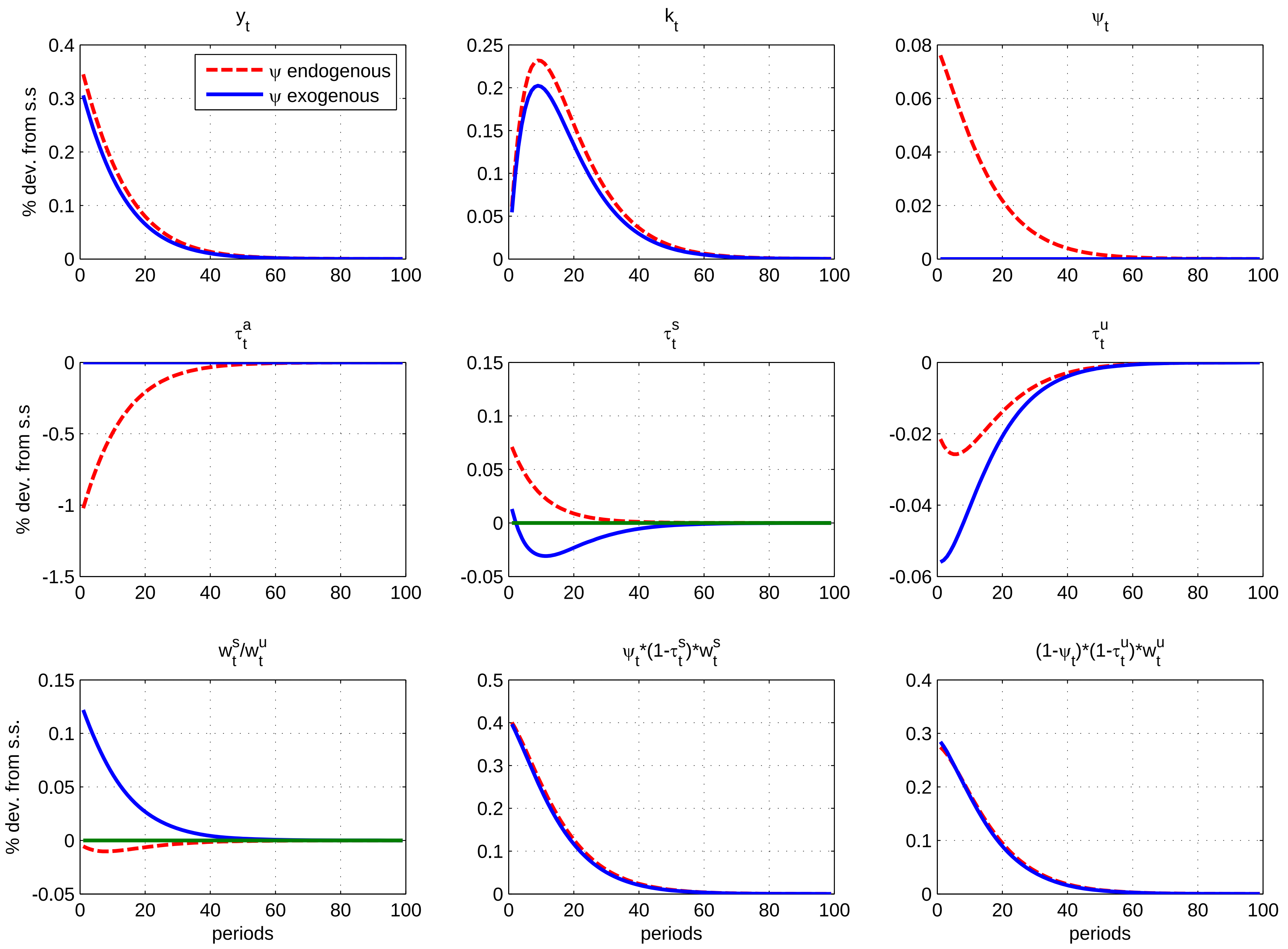

Figure 4: Impulse responses to 1% temporary shock to goverment spending