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Growth and scale effects: the role of knowledge spillovers

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Abstract

In recent two-R&D-sector growth models, scale effects are removed and the endogeneity of long-run growth is preserved. However, once knowledge spillovers across different types of R&D are introduced, long-run growth ceases to be endogenous. Moreover, increasing the dimension of technological progress reinforces the generality of the conclusion.

Keywords: Endogenous growth; Semi-endogenous growth; R&D; Technological progress; Knowledge spillover

JEL classification: O30

1. Introduction

The major drawback of the first-generation R&D-based growth models (e.g., Aghion and Howitt, 1992; Grossman and Helpman, 1991; Romer, 1990) is the scale effect prediction that a larger economy grows faster. To overcome this empirically unsupported property (see Jones, 1995a), Jones (1995b) proposed an alternative model which exhibits 'semi-endogenous' growth. Semi-endogenous growth basically means that (i) technological change is endogenous in the sense that it requires real resources, but (ii) long-run growth is exogenous as in the neo-classical growth models.[1]

In response to Jones' critique, several models have recently been put forward, which preserve endogeneity of R&D-driven growth.[2] These models have a common basic structure in which technological advance occurs on two dimensions, namely, quality and variety of goods. The key results of these studies are (i) semi-endogenous growth is limited to one-R&D-sector models, and (ii) long-run growth can still be endogenous without scale effects in the two-R&D-sector framework.

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[1]The same properties are also found in Kortum (1997) and Segerstrom (1998).

[2]For example, Aghion and Howitt ((1998), Chapter 12), Howitt (1999) and Young (1998). See Jones (1999) for more Refs. therein.

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The first purpose of this note is to show that these key results from two-R&D-sector models are overturned once knowledge spillovers between the two types of R&D are introduced. Such externalities are largely neglected in the studies referred to above, despite the fact that inter-industry knowledge spillovers are widely held to be substantial (e.g., see Griliches, 1995, Table 3.4, p. 72). Using a reduced form model, we will show that R&D-driven growth ceases to be endogenous in the long run under very mild conditions. Specifically, we show that endogenous growth requires two 'knife-edge' conditions, and growth is in general semi-endogenous unless a double coincidence of parameter values occurs. The microfoundations of the model presented are fully developed in Li (2000).

Second, and more importantly, these results are generalized, assuming that technology advances in a k-dimensional space where $k \geq 2$. A higher dimension of technological progress is not only general, but also arguably more realistic, given the multifarious nature of the modern technological innovation. The key result is that if growth is to be endogenous, at least k knife-edge conditions should be satisfied. That is, as the dimensions, k, of technological progress increases, it becomes progressively more difficult for long-run growth to be endogenous, and the generality of semi-endogenous growth is reinforced. Thus, Jones' (1995b) one-R&D-sector model of semi-endogenous growth is more general than originally thought.

Following Jones (1999), analysis is conducted in a reduced form without introducing microfoundations on the allocation of resources to R&D. Doing so is sufficient to demonstrate our key results, as establishing semi-endogenous growth effectively does not require detailed microfoundations. That is, our main results hold whatever the story behind the determination of the resource allocation between manufacturing and research activities.[3]

After describing the models and our key results, some further implications will be discussed towards the end of this article. We conclude that further research is required to fully establish whether R&D-driven growth is endogenous or semi-endogenous *in general*.

2. A two-R&D-sector model

In line with the recent literature, we first consider an economy where technological progress takes the dual form of improving quality and expanding the variety of goods. To simplify notation, time subscripts are suppressed unless ambiguity may arise. There are two sectors of production: manufacturing (physical goods) and R&D (intangible goods). Both production activities require labor, which is the only production factor. L denotes working population which grows at a given rate of $\lambda > 0$.

We use r to denote the proportion of workers employed in R&D activities, and the complementary fraction $1 - r$ of workers are used in manufacturing. Furthermore, a fraction m of rL workers is devoted to R&D aimed at expanding a variety of innovative goods, and the remaining proportion $n = 1 - m$ of rL researchers engage in quality-improving R&D. Parameters r and m (hence n) are assumed to be constant, which allows us to side-step the details of the microfoundations required to endogenize them. We are not interested in stories which endogenize r and $m = 1 - n$, but rather in whether long-run growth is a function of these parameters. Thus, the criterion adopted in this note is

[3]In principle, the framework used in Li (2000) can be used to develop a k-R&D-sector model.

that growth is endogenous if it is affected by r and/or $m = 1 - n$, and it is semi-endogenous otherwise. This captures, in a simple way, the essential differences between two types of long-run growth driven by endogenous technological change.

The production function for final output Y is given by

$$Y = \left[\int_0^v (q_j x_j)^\alpha \right]^{1/\alpha} \tag{1}$$

where x_j denotes intermediate goods of quality q_j, and the number of varieties is given by v. As regards production of inputs, producing one unit of x_j requires one worker, so that $(1 - r)L = \int_0^v x_j \, dj = vx$. The second equality uses the fact that $x_j \equiv x$ for all j in equilibrium, given the symmetric nature of the model. For simplicity, we assume $q_j \equiv q$ for all j. Then, the production function (1) is reduced to

$$Y = v^{\frac{1-\alpha}{\alpha}} q(1 - r)L \tag{2}$$

This shows that the aggregate level of technology is determined by the number of varieties of goods and their quality. That is, technology advances on two dimensions v and q. From (2), it should be clear that output per worker $y \equiv Y/L$ grows at a rate of

$$g_y = \frac{1 - \alpha}{\alpha} g_v + g_q \tag{3}$$

where g_χ is the growth rate of a variable χ. Growth will be semi-endogenous if and only if both g_v and g_q are independent of r and $m = 1 - n$, and are pinned down by λ, whereas it will be endogenous if at least one of the component growth rates is affected by r and/or m.

Turning to the dynamics of knowledge accumulation, the production function of new varieties of goods is given by

$$\dot{v} = \frac{mrLK_v}{q} \quad \text{where} \quad K_v = v^{\phi_v} q^{\delta_v}, \quad \phi_v, \delta_v > 0. \tag{4}$$

This technology exhibits CRS in workers used, mrL, and the term K_v captures externality effects in research. v^{ϕ_v} represents knowledge spillovers within variety R&D, and q^{δ_v} reflects inter-R&D knowledge spillover effects from quality R&D. The denominator q reflects the fact that the latest variety has the initial quality q, and its invention becomes progressively difficult as q gets larger.[4]

As regards quality improvement, it is modelled as a continuous process rather than a series of discrete jumps.[5] Thus, technology progresses on the quality dimension according to

$$\dot{q} = \frac{nrLK_q}{v} \quad \text{where} \quad K_q = v^{\phi_q} q^{\delta_q}, \quad \phi_q, \delta_q > 0. \tag{5}$$

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[4] q in the denominator is purely for expositional purposes. Our key results do not depend on this assumption. Moreover, the exponent of q, which is one, can be easily generalized without any gain of insight.

[5] This is purely for expositional purposes, and the 'quality-ladder' framework can be introduced into a microfounded model. See Li (2000).

Quality improves in all varieties, and quality R&D in each variety uses nrL/v workers. K_q represents knowledge spillovers within and between R&D activities. The inter-R&D knowledge spillover effects that are stressed in this note are captured by q^{δ_v} in (4) and v^{ϕ_q} in (5). The extent of such externalities are measured by δ_v and ϕ_q.

To examine the long-run behavior of the model, let us divide both sides of the first Eqs. of (4) and (5) to obtain

$$g_v = \frac{mrL}{v^{1-\phi_v} q^{1-\delta_v}}, \quad g_q = \frac{nrL}{v^{1-\phi_q} q^{1-\delta_q}}. \tag{6}$$

Note that the balanced growth path requires that both g_v and g_q are constant. That is, the denominator and numerator must grow at the same rate in each equation of (6). This fact leads to the system of two equations with two unknowns g_v and g_q:

$$\lambda = (1-\phi_v)g_v + (1-\delta_v)g_q, \quad \lambda = (1-\phi_q)g_v + (1-\delta_q)g_q. \tag{7}$$

These equations must be satisfied along the balanced growth path irrespective of whether growth is endogenous or semi-endogenous. First, note that $g_v = g_q = 0$ cannot occur for positive population growth $\lambda > 0$. Second, a unique solution exists if the equations are linearly independent, which requires that the determinant of the coefficient matrix of (7) is non-zero, i.e.

$$D \equiv (1-\phi_v)(1-\delta_q) - (1-\phi_q)(1-\delta_v) \neq 0. \tag{8}$$

Solving the system (7), we have

$$g_v = \frac{\delta_v - \delta_q}{D}\lambda, \quad g_q = \frac{\phi_q - \phi_v}{D}\lambda. \tag{9}$$

It can be easily verified that the numerators have the same sign as D. Note that g_v and g_q are independent of r and $m = 1 - n$. They are determined by population growth λ and parameters that measure the extent of knowledge spillovers. The same feature can be found in the one-R&D-sector models of semi-endogenous growth, e.g., Jones (1995b). Note that this result is obtained purely from the R&D equations and does not require any information of private incentives for R&D and consumer preferences. Thus,

Result 1. Long-run growth is semi-endogenous as long as $D \neq 0$.

Note that this result is valid irrespective of whether or not r and $m = 1 - n$ are endogenous, vindicating our reduced-form approach.

The intuition behind this result is similar to that of one-R&D-sector models. Initially, suppose $\lambda = 0$ and $\phi_v = \phi_q = \delta_v = \delta_q = 1$. Then Eqs. (4) and (5) become $g_v = nrL$ and $g_q = mrL$, in which case we have endogenous growth with scale effects. One way of eliminating scale effects is to assume limited knowledge spillovers, i.e., ϕ_v, ϕ_q, δ_v, $\delta_q < 1$. In this case, however, long-run growth is not sustained in the absence of population growth, since the denominators in (6) get larger and larger. Sustained growth is possible only when $\lambda > 0$. This fact leads to semi-endogenous growth. Note that the equilibrium does not require parameter restrictions of ϕ_v, ϕ_q, δ_v, $\delta_q < 1$. In fact, some of them can be larger than one, as long as (7) and (8) are satisfied. For example, $\phi_v > 0$ and $\delta_v < 1$ are permissible.

Next, let us turn to the case of $D = 0$. Equations in (9) are no longer well-defined, as the system of equations (7) are linearly dependent. For the equations to be consistent, $D = 0$ occurs when two parameter restrictions $\phi \equiv \phi_v = \phi_q$ and $\delta \equiv \delta_v = \delta_q$ are imposed, since (7) is the nonhomogeneous system (i.e., $\lambda > 0$). An important consequence of these two knife-edge conditions is that equations (7) are reduced to

$$\lambda = (1 - \phi) g_v + (1 - \delta) g_q. \tag{10}$$

Since there are two unknowns g_v and g_q in a single equation, their values cannot be determined by (10) alone. This fact sets the stage for endogenous growth. To pin down g_v and g_q, we need another equilibrium condition that will be based on private R&D incentives and consumer preferences. We cannot go further than this because no microfoundations explaining how much resources are allocated to each type of R&D are present in the model. However, it should be apparent that g_v and g_q will be functions of r and $m = 1 - n$ in this case.[6] Now we are in a position to state

Result 2. Long-run growth is endogenous when $D = 0$, which in turn requires two knife-edge conditions $\phi_v = \phi_q$ and $\delta_v = \delta_q$.

Note that $\phi_v = \phi_q$ and $\delta_v = \delta_q$ means $D = 0$, whereas $\phi_v \neq \phi_q$ or/and $\delta_v \neq \delta_q$ implies $D \neq 0$. Thus, $D = 0$ is far more restrictive than $D \neq 0$, because $\phi_v = \phi_q$ and $\delta_v = \delta_q$ are obtained on a measure-zero subset of relevant parameter values. In this sense, semi-endogenous growth is more general than endogenous growth.

To highlight the crucial role of inter-R&D spillovers, let us consider recent two-R&D-sector models of endogenous growth. Following a survey of Jones (1999), we capture the essential features of this class of models by assuming $\phi_v = \phi_q = 0$ and $\delta_v = \delta_q = 1$. Note that $\phi_q = 0$ means that one channel of inter-R&D spillovers is precluded. The R&D Eqs. (4) and (5) now become

$$\dot{v} = mrL, \tag{11}$$

$$\dot{q} = \frac{nrLq}{v}. \tag{12}$$

From (11), we have $g_v = mrL/v$, which implies $g_v = \lambda$ along the balanced growth path. Moreover (12) gives $g_q = nrL/v$ which is constant due to $g_v = \lambda$. Further, it can be rearranged into $g_q = nrg_v/mr = \lambda n/m$. This confirms that growth can be endogenous. However, this result is not surprising, since it is built on the two knife-edge restrictions $\phi_v = \phi_q$ and $\delta_v = \delta_q$, identified above.[7]

[6] See Li (2000) for details of microfoundations.

[7] Readers may have realized similarity of our reasoning to Mulligan and Sala-i-Martin (1993) who consider a two-sector model of physical and human capital accumulation. They identify two knife-edge conditions for a constant endogenous growth rate, which, if violated, leads to unsteady growth even in the long run. By contrast, in our model the long-run growth rate is constant, whether or not two knife-edge requirements are met.

3. A *k*-R&D-sector model

This section generalizes the model by increasing the number of dimensions of technological progress. We assume that each variety good has $k-1$ ($k=2,3,\ldots$) quality attributes which can be improved through $k-1$ different quality R&D activities. Computers provide an obvious example. Their performance can be enhanced with faster chips, larger memory and hard disks, flat screens, etc. We use n^i to denote the proportion of rL workers used in R&D which improves the ith quality attribute, so that $n^1+\cdots+n^{k-1}=1-m$.

To capture this multi-dimensional technological progress, the production function is assumed to take the form of

$$Y=\left[\int_0^v\left(q_j^1 q_j^2\cdots q_j^{k-1}x_j\right)^{\alpha}\right]^{1/\alpha} \tag{13}$$

where q_j^i denotes the ith quality attribute of variety j. Technology advances on k-dimensions of variety and $k-1$ quality attributes. Given $x_j\equiv x$ and $q_j^i\equiv q^i$ for all j, the production function (13) implies that output per capita grows at

$$g_y=\frac{1-\alpha}{\alpha}g_v+g_q^1+g_q^2+\cdots+g_q^{k-1} \tag{14}$$

where g_q^i is the growth rate of q^i. Thus, semi-endogenous growth occurs if all of g_v and g_q^i, $i=1,\ldots k-1$, are independent of r, m and n^i and pinned down by population growth. Endogenous growth occurs if at least one of them is affected by r, m and/or n^i.

New varieties and higher qualities are created according to

$$\dot{v}=\frac{mrLK_v}{\prod_{i=1}^{k-1}q^i}\quad\text{where}\quad K_v=v^{\phi_v}\prod_{i=1}^{k-1}(q^i)^{\delta_v^i},\quad \phi_v,\delta_v^i>0, \tag{15}$$

$$\dot{q}^i=\frac{n^i rLK_q^i}{v\prod_{i'=1}^{k-1}q^{i'}/q^i}\quad\text{where}\quad K_q^i=v^{\phi_i}\prod_{i'=1}^{k-1}(q^{i'})^{\delta_i^{i'}},\quad \phi_i,\delta_i^{i'}>0,\ i=1,\ldots k-1. \tag{16}$$

These are an extended version of (4) and (5). For variety R&D, parameters δ_v^i measure inter-R&D spillovers from $k-1$ different types of quality innovations. For the ith quality R&D, the degree of inter-R&D spillovers are measured by ϕ_i and $\delta_i^{i'}$, $i'\neq i$. As before, we assume that difficulty of invention rises with the degree of sophistication of the product, and it is captured by $\prod_{i=1}^{k-1}q^i$ for variety R&D and $\prod_{i'=1}^{k-1}q^{i'}/q^i$ for the ith quality R&D.

Eqs. (15) and (16) imply that the rates of technological progress on variety and $k-1$ quality dimensions are given by

$$g_v=\frac{mrL}{v^{1-\phi_v}\prod_{i=1}^{k-1}(q^i)^{1-\delta_v^i}},\qquad g_q^i=\frac{n^i rLK_q^i}{v^{1-\phi_i}\prod_{i'=1}^{k-1}(q^{i'})^{1-\delta_i^{i'}}},\qquad i=1,..k-1. \tag{17}$$

As before, we focus on the balanced growth path. Since g_v and g_q^i must be constant along this path, we end up with the system of k equations with k unknowns:

$$\lambda = (1-\phi_v)g_v + \sum_{i=1}^{k-1}(1-\delta_v^i)g_i \tag{18}$$

$$\lambda = (1-\phi_i)g_v + \sum_{i'=1}^{k-1}(1-\delta_i^{i'})g_{i'}, \quad i=1,\dots k-1. \tag{19}$$

If all these equations are linearly independent, we can solve for g_v and g_q^i, $i=1,\dots k-1$. That is, the rates of progress on all technological dimensions are solely determined by population growth and parameters which govern the degree of externalities within and across R&D activities. This result can be formally stated as follows:

Result 3. Long-run growth is semi-endogenous when the rank of D_k is k where D_k is the $k \times k$ coefficient matrix of (18) and (19).

Next let us search for conditions which give rise to endogenous growth. Note that growth will be endogenous when at least one of g_v and g_q^i, $i=1,\dots k-1$, is affected by r, m or/and n^i, that is, when at least one of g_v and g_q^i cannot be determined in the system of (18) and (19). Suppose that g_q^i, $i=1,\dots k-1$, are determined but g_v is not. Then, the row associated with (18) in D_k must be a linear combination of another row, say the row corresponding to Eq. (19) for $i=i'$, so that the rank of D_k is $k-1$. This happens if k knife-edge conditions $\phi_v = \phi_{i'}$ and $\delta_v^i = \delta_{i'}^i$, $i=1,\dots k-1$, are satisfied, given the non-homogeneous system. A similar reasoning can be applied to the case where two of g_v and g_q^i are not determined in the system. In this case, the rank of D_k must be $k-2$, and a total of $2k$ knife-edge conditions are required for endogenous growth. Similarly, for all of g_v and g_q^i to be endogenously determined, we require a total of k^2 knife-edge conditions. Thus, we have the following result:

Result 4. Long-run growth is endogenous when the rank of D_k is equal to or less than $k-1$. Thus, the minimum requirements for endogenous growth are k knife-edge conditions.

Note that the requirements for the rank of D_k being $k-1$ are far more restrictive than the ones for full rank, since the former needs k knife-edge conditions. Moreover, as the dimension of technological progress increases, the number of knife-edge requirements increases. In this sense, increasing the dimension of innovation, which arguably makes the model more realistic, strengthens the generality of semi-endogenous growth.

4. Discussion

The key message of this paper is that inter-R&D knowledge spillovers render semi-endogenous growth more likely than endogenous growth, and this result strengthens as the dimension of technological progress expands. These results reinforce the argument of Jones (1995b) that growth is semi-endogenous in a one-R&D-sector model, and highlight the limits of various attempts at responding to Jones's critique on the basis of two-dimensional technical innovation.

However, it seems premature to conclude that R&D-driven growth is semi-endogenous *in general*. There still remain further questions unanswered. Recall that the spillover parameters (e.g., ϕ_v, ϕ_q, δ_v, δ_q in the two-R&D-sector model) which play a crucial role in our analysis are assumed exogenous. Although this assumption is a plausible starting point, there is no reason why they must be independent of economic incentives. In fact, there is empirical evidence indicating that those spillover parameters are endogenous to some extent. In their important study, Cohen and Levinthal (1989) show that R&D has 'two faces' in the sense that it not only creates new knowledge but also improves the ability of the firms to absorb knowledge created elsewhere. That is, knowledge spillovers are not automatic, but require deliberate investment, e.g., training and research facilities. To the extent that firms recognize this (which seems plausible) and try to improve their 'absorptive capacity' to create new knowledge, the spillover parameters are endogenous.

Viewed this way, one legitimate question is ''Are there any mechanisms which endogenously generate the knife-edge conditions required for endogenous growth?'' To answer this question, Acemoglu (2001) may be a useful starting point. He studies how economic incentives direct the *production* of knowledge in the presence of two types of knowledge. This idea is relevant in understanding the *use* of knowledge, i.e., how economic incentives direct research firms' efforts to enhance their absorptive capacity. There might (or might not) be economic forces which lead R&D activity towards knife-edge conditions. Answering this question is clearly important in our understanding of long-run growth which is largely driven by technical progress.

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