A Note on Agenda Restrictions in Multi-issue Bargaining

by

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Abstract

We study the effects of a *between-cake* discount factor in the agenda selection problem faced by a multi-issue committee. The presence of an interval of time between bargaining stages is a reasonable and realistic assumption. We show that this assumption simplifies the agenda selection problem strongly. In particular, the equilibrium multiplicity obtained in In and Serrano (2002) does not hold: a unique SPE can be established among the sequential bargaining procedures.

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Key words: bargaining, agenda, Subgame Perfect Equilibrium

1 Introduction

In general, game-theoretical attempts to solve the agenda selection problem have allowed multi-issue committees to select not only sequential but also simultaneous procedures (e.g., Busch and Horstmann, 1997, 1999, Inderst, 2000, Lang and Rosenthal, 2001, In and Serrano, 2003, in the context of games featuring perfect information). Moreover, when the agenda selection problem is fully endogenised, it has been argued that a simultaneous procedure should be chosen since this allows the exploitation of all trading opportunities (e.g., Inderst, 2000 and In and Serrano, 2003). However, simultaneous procedures are not very common in practice, since parties may be unable to discuss more than one issue at a time. A notable exception is In and Serrano, 2002 (I-S, henceforth), where the authors endogenise the agenda selection problem and restrict their attention to the more plausible case of issue-by-issue procedures. They show that in this case multiple equilibria, possibly with delays in reaching an agreeement, can arise. This is not very surprising since even in a framework based on an exogenous issue-by-issue agenda there is often no consensus over the best agenda (in particular, parties are either indifferent among agendas or they have different preferences over agendas). However, it can be shown that this is not always the case (see Flamini, 2002, for a discussion).

We adopt a standard framework here, similar to I-S, but where we introduce the possibility of an interval of time between bargaining stages, where in each stage parties bargain over the division of a surplus *à la* Rubinstein. This assumption is reasonable and realistic, since in virtually any decision-making process, parties take some time to switch the focus from one item to another one. The introduction of this interval of time also has important consequences for the agenda selection problem. The indeterminacy of equilibria shown in I-S no longer applies. There is a unique equilibrium payoff that each player obtains in the agenda selection problem. Moreover, such an equilibrium payoff can only be obtained by choosing the agenda that sets the most important issue first.

In the next section, we briefly present the model. To make the comparison more straightforward, we use the same notation as in I-S, whenever possible. In section 2.1, we solve the game and show the results. Section 3 concludes the paper.

2 The model

Two parties, named 1 and 2, bargain over the agreement on two issues (or the division of two cakes), also named 1 and 2. At each bargaining stage, parties attempt to divide a cake as in the classic Rubinstein model (1982). Only after an agreement has been reached can parties start to negotiate over the second issue. The first mover is player 1. A proposal is a division $(x_k, 1 - x_k)$ over cake k, with $k = 1, 2$, where x_k is player 1's share, $0 \leq x_k \leq 1$. If player i makes a successful proposal, player j will propose next, with $i, j = 1, 2$ and $i \neq j$.

We assume that there is an interval of time not only between a rejection and a new proposal, say Δ, but also between an acceptance and a new proposal, say τ (with $\tau \geqslant \Delta$)[1]. To simplify players have the same time preferences[2], represented by the rate of time preference r. There are therefore two types of discount factor, a *between-cake* discount factor $\alpha = \exp(-r\tau)$ which applies between bargaining stages and a *within-cake* discount factor $\delta = \exp(-r\Delta)$, which applies between rounds within a bargaining stage.

The implementation of the agreement is sequential. Moreover, players' utilities are linear in the shares they obtain. If players reach an agreement on cake 1 first and then on cake 2 without delay then their utilities are as follows:

$$u_1 = ax_1 + \alpha x_2 \tag{1}$$

$$u_2 = b(1 - x_1) + \alpha(1 - x_2) \tag{2}$$

where a and b represents the relative importance of cake 1 to player 1 and 2 respectively. As usual, in disagreement parties get zero (at least in that stage). To simplify the analysis we focus on the case in which cake 1 represents the most important issue

[1] This time framework has been introduced by Muthoo (1995, 1999). His focus is on repeated games and therefore an infinite number of cakes (of constant size) are considered. In our framework, the focus is on the agenda selection problem and for this reason a finite number of cakes (of different sizes) is analysed.

[2] This is a common but not innocuous assumption. For a discussion on the effect of different time preferences on the agenda formation problem, see Flamini (2002).

(in particular, $a > b \geq 1$)[3]. Similar results can be obtained when cake 2 is the most important. However, when there is no consensus over the importance of the issues, for the same mechanism explained in I-S, multiple equilibria can arise. This is also consistent with the fact that parties have different preferences over agendas when these are exogenously given.

2.1 The Equilibrium

The focus is on subgame perfect equilibria (SPE). Let G_i (F_i) be the set of possible utilities when player 1 (2) chooses to offer an arbitrary division on cake i first and this is accepted, with $i = 1, 2$. Since at the second stage the SPE is given by the Rubinsteinian shares, then,

$$G_i = \{(u_1, u_2) \in R^2 : x_i \in [0,1], x_j = \frac{\delta}{1+\delta}\}, \quad (3)$$

$$F_i = \{(u_1, u_2) \in R^2 : x_i \in [0,1], x_j = \frac{1}{1+\delta}\} \quad (4)$$

with $i, j = 1, 2$ and $i \neq j$. These can also be written as follows,

$$G_1 = \{u_1 = -\frac{a}{b}u_2 + a + \frac{a}{b}\alpha - \frac{\delta\alpha(a-b)}{(1+\delta)b}, u_2 \in [\alpha\frac{1}{1+\delta}, b + \alpha\frac{1}{1+\delta}]\} \quad (5)$$

$$G_2 = \{u_1 = 1 - u_2 + \alpha b + \frac{\delta\alpha(a-b)}{1+\delta}, u_2 \in [\alpha\frac{b}{1+\delta}, 1 + \alpha\frac{b}{1+\delta}]\} \quad (6)$$

[3] Similar results can be obtained when $1 \leq a < b$. When $a = b > 1$, the analysis is straightforward and omitted. Note that in this case, as in I-S, there is a unique SPE payoff. However, differently from I-S, where parties are indifferent between bargaining procedures, in our framework parties strictly prefer the agenda that sets the most important issue first.

$$F_1 = \{u_2 = -\frac{b}{a}u_1 + b + \alpha - \frac{\alpha(a-b)}{(1+\delta)a}, u_1 \in [\alpha\frac{1}{1+\delta}, a + \alpha\frac{1}{1+\delta}]\} \quad (7)$$

$$F_2 = \{u_2 = -u_1 + 1 + \alpha b + \frac{\alpha(a-b)}{1+\delta}, u_1 \in [\alpha a\frac{1}{1+\delta}, 1 + \alpha a\frac{1}{1+\delta}]\} \quad (8)$$

Let $g(u_2)$ (and $f(u_1)$) be the highest possible utility that player 1 (2, respectively) can get without delay when he proposes first and given that the other player gets u_2 (u_1, respectively). Then we can show the following result.

Lemma 1 *For $a > b > b^u = \frac{a(1+\delta(1-\alpha))}{(a(1-\alpha)(1+\delta)+\alpha)} \geq 1$, the functions $g(u_2)$ and $f(u_1)$ are concave and continuous, in particular,*

$$g(u_2) = \begin{cases} a + \alpha\frac{\delta}{1+\delta} & \text{if } u_2 \in [0, \alpha\frac{1}{1+\delta}] \\ u_1 \text{ in } G_1 & \text{if } u_2 \in [\alpha\frac{1}{1+\delta}, b + \alpha\frac{1}{1+\delta}] \end{cases} \quad (9)$$

$$f(u_1) = \begin{cases} b + \alpha\frac{\delta}{1+\delta} & \text{if } u_1 \in [0, \alpha\frac{1}{1+\delta}] \\ u_2 \text{ in } F_1 & \text{if } u_1 \in [\alpha\frac{1}{1+\delta}, a + \alpha\frac{1}{1+\delta}] \end{cases} \quad (10)$$

Proof. We first focus on $g(u_2)$. Differently from I-S, $g(u_2)$ cannot include u_1 defined in G_2. Indeed, u_1 defined in G_1 is larger than u_1 defined in G_2 if and only if $u_2 < u_2^U$, where

$$u_2^U = \frac{b}{a-b}(a(1+\frac{\alpha}{b}) - (1+\alpha b)) - \frac{\delta\alpha}{1+\delta}(b+1) \quad (11)$$

However, u_2^U is larger than the upper bound for u_2 in G_2 (that is, $1 + \alpha\frac{b}{1+\delta}$) for b sufficiently large (i.e., $b > \frac{a(1+\delta-\alpha))}{(a(1-\alpha)(1+\delta)+\alpha\delta)}$ which is always satisfied when $b > b^u$). Note that the function $g(u_2)$ cannot include the corner solution in which player 1 gets the entire cake 2 at the first stage and then parties share the following cake as in the

Rubinstein bargaining model (i.e., player 1's payoff would not be the highest possible in this scenario).

Similarly, it can be shown that for $b > b^u$, u_2 defined in F_1 is larger than both u_2 defined in F_2 and $u_2 = 1 + \alpha b \frac{\delta}{1+\delta}$ (that is, the payoff in the case in which player 2 obtains the entire cake 2 and then share cake 1 as in the Rubinstein bargaining game). ■

Since the multiplicity of equilibria shown in I-S are based on the non-concavity of the functions $g(u_2)$ and $f(u_1)$, it is clear that Lemma 1 has an important consequence on the outcome of the game.

Proposition 2 *Let $a > b > b^u \geq 1$, then the unique SPE is to play according to the agenda that sets the most important issue first.*

Proof. The proof consists in showing that the solution of the system $u_1 = \delta g(u_2)$ and $u_2 = \delta f(u_1)$ is unique. Given the definition of $g(u_2)$ and $f(u_1)$, the two equations $u_1 = \delta g(u_2)$ and $u_2 = \delta f(u_1)$ lead to four systems. However, only one has a feasible solution. This is as follows,

$$\begin{cases} u_1 = \delta(-\frac{a}{b}u_2 + a + \frac{a}{b}\alpha - \frac{\delta\alpha(a-b)}{(1+\delta)b}), & \text{if } u_2 \in [\alpha\frac{1}{1+\delta}, b + \alpha\frac{1}{1+\delta}] \\ u_2 = \delta(-\frac{b}{a}u_1 + b + \alpha - \frac{\alpha(a-b)}{(1+\delta)a}), & \text{if } u_1 \in [\alpha\frac{1}{1+\delta}, a + \alpha\frac{1}{1+\delta}] \end{cases} \tag{12}$$

The unique solution of this system is given by (u_1^*, u_2^*) defined below,

$$u_1^* = \frac{a(b+\alpha)\delta}{b(1+\delta)} \text{ and } u_2^* = \frac{b(a+\alpha)\delta}{a(1+\delta)} \tag{13}$$

Moreover, this solution is feasible since both u_i^* with $i = 1, 2$ belongs to the interval of interest defined in the system (12). Indeed, $u_1^* > \frac{\alpha}{1+\delta}$ if and only if

$$\delta\left(\frac{a\alpha}{b} + a\right) > \alpha \tag{14}$$

and this is always satisfied since $\delta > \alpha$ and the bracketed expression in (14) is larger than 1. Moreover, $u_1^* < a + \frac{\alpha}{1+\delta}$, since $a(b - \alpha\delta) + \alpha b > 0$. Similarly, $u_2^* > \frac{\alpha}{1+\delta}$ if and only if $a(b\delta - \alpha) + \alpha b\delta > 0$, which is always satisfied. Finally, $u_2^* < b + \frac{\alpha}{1+\delta}$ since the expression $b(a - \alpha\delta) + \alpha a$ is always positive. Instead, for the remaining three systems the solution is not feasible (i.e., the constraints are not satisfied, see Appendix).

Given the unique solution (u_1^*, u_2^*) to the system $u_1 = \delta g(u_2)$ and $u_2 = \delta f(u_1)$, there is a unique SPE in the bargaining game. This is reached without delay and the equilibrium utilities are $(\frac{u_1^*}{\delta}, u_2^*)$. Therefore, parties must play according to the agenda that sets the most important issue first. ■

In other words, the indeterminacy of equilibria obtained in I-S does not hold in general. Parties play according to the agenda that sets the most important issue first. Our result is based on the assumption that the relative importance of cake 1 to player 2 is relatively high (i.e., $b > b^u$). Together with $a > b \geq 1$, this implies that the between-cake discount factor, α, although it can be high, has to be strictly smaller than 1 (indeed, for $\alpha = 1$, $b^u = a$).

When the assumption on the relative importance of issue 1 to player 1 (i.e., $b > b^u$) is relaxed, there are two possible cases: either $b < \frac{a(1+\delta-\alpha))}{(a(1-\alpha)(1+\delta)+\alpha\delta)} < b^u$ or

$\frac{a(1+\delta-\alpha))}{(a(1-\alpha)(1+\delta)+\alpha\delta)} < b < b^u$. In the former, the functions $g(u_2)$ and $f(u_1)$ are non-concave and $g(u_2)$ is also discontinuous (see the proof of lemma 1). Then, Proposition 2 cannot hold and, for the same mechanism explained in I-S, a multiplicity of equilibria can arise. In the second case, the function $g(u_2)$ is non-concave and discontinuous, while $f(u_1)$ remains as defined in (10) (see the proof of lemma 1). This implies that under certain conditions it can still be possible to obtain a unique solution as in Proposition 2. However, when the within-cake discount factor tends to 1 ($\delta \to 1$), the interval $b^u - \frac{a(1+\delta-\alpha))}{(a(1-\alpha)(1+\delta)+\alpha\delta)}$ becomes degenerate (tends to a point). Therefore, analysing this case is not particularly interesting since it is very small when the frictions tend to disappear.

3 Conclusions

In this paper, we modify the I-S set-up by assuming the existence of an interval of time between an acceptance and a new proposal. This is a reasonable and realistic assumption with important consequences to the agenda selection problem: the agenda selection problem can have a unique solution when parties are restricted to negotiate under issue-by-issue procedures. We therefore show, first, the conditions under which the equilibria multiplicity shown in I-S holds and, second, when there is a unique solution of the game (in particular, a unique equilibrium payoff), parties select an agenda uniquely and this is the one that sets the most important issue first.

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Appendix

Remaining part of the Proof of Proposition 2. In the main text, we show that one of the four systems which can be derived by the set of equations, $u_1 = \delta g(u_2)$ and $u_2 = \delta f(u_1)$, has a feasible solution (that is, all the constraits are satisfied), see (12). In this remaining part of the proof we show (i) why the three remaining systems do not have a feasible solution and (ii) that for the unique (feasible) solution, parties play according to the agenda that sets the most important issue first.

(i) The first of the three remaining systems that can be derived from $u_1 = \delta g(u_2)$ and $u_2 = \delta f(u_1)$ is as follows,

$$\begin{cases} u_1 = \delta(a + \alpha\frac{\delta}{1+\delta}) \text{ with } u_2 \in [0, \alpha\frac{1}{1+\delta}] \\ u_2 = \delta(-\frac{b}{a}u_1 + b + \alpha - \frac{\alpha(a-b)}{(1+\delta)a}) \text{ with } u_1 \in [\alpha\frac{1}{1+\delta}, a + \alpha\frac{1}{1+\delta}] \end{cases} \tag{15}$$

In this case, the intersection is given by u_1 defined in (15) and

$$u_2 = \frac{\delta(\alpha a\delta + b(a+\alpha)(1-\delta^2))}{(1+\delta)a} \tag{16}$$

The latter is clearly positive, however, it is not larger than $\alpha\frac{1}{1+\delta}$ iff

$$\delta b(a+\alpha) < \alpha a \tag{17}$$

But this cannot be satisfied since $\delta b > \alpha$ and $a + \alpha > a$.

The second of the three remaining system which can be derived from the set of equations $u_1 = \delta g(u_2)$ and $u_2 = \delta f(u_1)$ is as follows,

$$\begin{cases} u_1 = \delta(-\frac{a}{b}u_2 + a + \frac{a}{b}\alpha - \frac{\delta\alpha(a-b)}{(1+\delta)b}) \text{ with } u_2 \in [\alpha\frac{1}{1+\delta}, b + \alpha\frac{1}{1+\delta}] \\ u_2 = \delta(b + \alpha\frac{\delta}{1+\delta}) \text{ with } u_1 \in [0, \alpha\frac{1}{1+\delta}] \end{cases} \tag{18}$$

For this system the intersection is given by u_2 defined in (18) and

$$u_1 = \frac{\delta(\alpha\delta b + a(b+\alpha)(1-\delta^2))}{(1+\delta)b} \tag{19}$$

This solution cannot be feasible since u_1 is larger than $\alpha\frac{1}{1+\delta}$ whenever,

$$\delta a(b+\alpha) > \alpha b \tag{20}$$

which is always satisfied, since $\delta > \alpha$, $(b+\alpha) > b$ and $a > 1$.

The final infeasible system is given by,

$$\begin{cases} u_1 = \delta(a + \alpha\frac{\delta}{1+\delta}) \text{ with } u_2 \in [0, \alpha\frac{1}{1+\delta}] \\ u_2 = \delta(b + \alpha\frac{\delta}{1+\delta}) \text{ with } u_1 \in [0, \alpha\frac{1}{1+\delta}] \end{cases} \tag{21}$$

Clearly, u_i is non-negative but cannot be smaller or equal to $\alpha\frac{1}{1+\delta}$ for $i = 1, 2$.

In conclusion, the only feasible solution to the system $u_1 = \delta g(u_2)$ and $u_2 = \delta f(u_1)$ is describe by (12) and is given by the pair (u_1^*, u_2^*) with

$$u_1^* = \frac{a(b+\alpha)\delta}{b(1+\delta)} \text{ and } u_2^* = \frac{b(a+\alpha)\delta}{a(1+\delta)} \tag{22}$$

while the equilibrium payoffs are given by $(\frac{u_1^*}{\delta}, u_2^*)$.

(ii) We now show that only the agenda that sets the most important issue first can be played in equilibrium. Since the second stage is played as the Rubinstein bargaining game, then, it is sufficient to show that according to the agenda that sets the most important issue first the following is satisfied,

$$\left(1-\left(\frac{u_1^*}{\delta}-\frac{\alpha\delta}{1+\delta}\right)\frac{1}{a}\right)b+\frac{\alpha}{1+\delta}=u_2^* \tag{23}$$

while according to the agenda that sets the less important issue first the following cannot be satisfied,

$$\left(1-\left(\frac{u_1^*}{\delta}-\frac{a\alpha\delta}{1+\delta}\right)\right)+\frac{\alpha b}{1+\delta}=u_2^* \tag{24}$$

If we substitute u_1^*, from (22), in (23) and (24), then (23) is always satisfied, while (24) can only be satisfied if

$$\frac{b(1+\delta)-a(b-\alpha)+\alpha b(b+\alpha\delta)}{b(1+\delta)}=\frac{b(a+\alpha)\delta}{a(1+\delta)} \tag{25}$$

This cannot hold in general (although it can in specific cases, for instance, when parties are indifferent between issues $a=b=1$ or, as in the set-up in I-S, $\alpha=1$ and $a=b$). ■